

**CORK INSTITUTE OF TECHNOLOGY  
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ**

**Autumn Examinations 2008/09**

**Module Title: Calculus & Statistics**

**Module Code:** MATH 6002

**School:** Science

**Programme Title:** Bachelor of Science in Applied Biosciences – Year 1  
Bachelor of Science in Analytical & Pharmaceutical Chemistry –  
Year 1

**Programme Code:** SBIOS\_7\_Y1  
SCHEM\_7\_Y1

**External Examiner(s):** Dr Paul Robinson

**Internal Examiner(s):** Ms H. Lordan, Ms. F. Wood

**Instructions:** Answer QUESTION 1 (compulsory - 30 marks) and  
TWO other questions (35 marks each)

**Duration:** 2 Hours

**Sitting:** Autumn 2009

**Requirements for this examination:** Mathematical Tables

**Note to Candidates:** Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.

Q1. Answer **each** of the following:

- (a) The length,  $l$  metres, of a metal rod at temperature  $\theta$   $^{\circ}\text{C}$  is given by  $l = 1.4 + 2 \times 10^{-4} \theta + 4 \times 10^{-6} \theta^2$ . Determine (i) the length and (ii) the rate of change of length  $l$ , when the temperature is  $100^{\circ}\text{C}$ . (5 marks)

- (b) Show that  $y = 3e^{2x}$  satisfies the equation  $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ .  
Show that  $y = e^{-x}$  is also a solution. (5 marks)

- (c) The gradient of a curve is given by  $\frac{dy}{dx} = 12x^3 - 5 + \frac{12}{x^3}$ . Find the equation of the curve if  $y = 40$  when  $x = 2$ . (5 marks)

- (d) Find the area enclosed by the curve  $y = 80e^{-0.4x}$  and the  $x$ -axis and the ordinates  $x = 1$  and  $x = 3$ . (5 marks)

- (e) The average age of a party of 12 people was 32. The average age increased by 3 years when three new people joined the party. What is the average age of the three new people? (5 marks)

- (f) Find the Median and 1<sup>st</sup> and 3<sup>rd</sup> Quartile values for this data:

25	23	63	16	39	21	41
15	34	54	42	11	28	56

(5 marks)

Q2.(a) Differentiate  $y = 3x - x^2$  from first principles.

(7 marks)

(b) Differentiate each of the following:

(i)  $\theta = 50 \sin(4t^2 - 0.8)$

(ii)  $y = \cos^2 x \cdot \ln(7x)$

(iii)  $s = \frac{e^{-t^2}}{(t^3 - t)}$

(16 marks)

(c) The function  $y = x^3 + x^2$  has two turning points. Locate each of these and identify minimum from maximum. Evaluate the  $y$  co-ordinates of each of the turning points and hence sketch the function.

(12 marks)

Q3.(a) Determine each of the following integrals:

(i)  $\int_1^2 \frac{3x^2 + 7x - 10}{x} dx$

(ii)  $\int (5x - 3)^4 dx$

(iii)  $\int_1^3 2xe^{x^2} dx$ .

(21 marks)

(b) Find the points at which the curve  $y = (x + 1)(x - 1)(x - 3)$  intersects the  $x$  axis. Sketch the curve. Use integration to find the area enclosed by the curve and the  $x$  axis.

(14 marks)

- Q4. A compound was analysed for manganese content. The table below shows the percentage content for a sample of 150 analyses:

Manganese Content %	Number of samples
26.0 – 26.4	19
26.5 – 26.9	25
27.0 – 27.9	33
28.0 – 28.9	44
29.0 – 29.9	15
30.0 – 30.9	10
31.0 – 32.9	4

- (i) Calculate the mean percentage manganese content ( $\bar{x}$ ) and the standard deviation ( $s$ ) from the mean. (16 marks)
- (ii) Construct a cumulative frequency table. (4 marks)
- (iii) Represent the information on a cumulative frequency polygon (ogive). (9 marks)
- (iv) Estimate the number of samples that lie in the range  $(\bar{x} - s)$  to  $(\bar{x} + s)$ . (6 marks)

# Standard Results of Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$	
$x^n$	$nx^{n-1}$	
$\ln x$	$\frac{1}{x}$	
$e^x$	$e^x$	
$e^{ax}$	$ae^{ax}$ $a = \text{constant}$	
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
$uv$	$u \frac{dv}{dx} + v \frac{du}{dx}$	... Product Rule
$\frac{u}{v}$	$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	... Quotient Rule

### Standard Integrals

$f(x)$	$\int f(x)dx$
$x^n$	$\frac{x^{n+1}}{n+1} \quad n \neq -1$
$\frac{1}{x}$	$\ln x$
$e^x$	$e^x$
$e^{ax}$	$\frac{1}{a} e^{ax} \quad a = \text{constant}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$

### Statistical Formulae:

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation } s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

#### *Assumed Mean Method:*

$$\text{Mean } (\bar{x}) = a + c \left( \frac{\sum f(d/c)}{\sum f} \right)$$

$$\text{Standard Deviation } s = c \sqrt{\frac{\sum f(d/c)^2}{\sum f} - \left( \frac{\sum f(d/c)}{\sum f} \right)^2}$$