

**CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ**

Semester 2 Examinations 2008/09

Module Title: Calculus & Statistics

Module Code: MATH 6002

School: Science

Programme Title: Bachelor of Science in Applied Biosciences – Year 1
Bachelor of Science in Analytical & Pharmaceutical Chemistry – Year 1

Programme Code: SBIOS_7_Y1
SCHEM_7_Y1

External Examiner(s): Dr Brendan O Regan
Internal Examiner(s): Ms H. Lordan, Ms. F. Wood

Instructions: Answer QUESTION 1 (compulsory - 30 Marks) and
TWO other questions (35 Marks each)

Duration: 2 Hours

Sitting: Summer 2009

Requirements for this examination: Mathematical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.

Q1. Answer **each** of the following:

(a) Differentiate: $y = 5x^2 + \frac{3}{2x} + 6\sqrt{x} - \ln(2x)$

(5 marks)

- (b) Find the co-ordinates of the turning point on the curve $y = 4x + e^{-x}$.
Determine whether the point is a maximum or a minimum turning point.

(5 marks)

(c) Find an expression for y given $\frac{dy}{dx} = 3x^2 - \frac{2}{3} + \frac{3}{x^2}$ and $y = 20$ when $x = 3$.

(5 marks)

(d) Evaluate $\int_0^{\pi/3} 3\sin(2x)dx$

(5 marks)

- (e) 21 samples have a mean mass of 24.2 mg, and 29 similar samples have a mean mass of 23.6mg. Determine the mean mass of the 50 samples.

(5 marks)

- (f) The number of hours lost due to illness for each of 12 semesters is as shown. Determine the median, and first and third quartiles for this data.

27	37	40	28	23	30
35	24	30	32	31	28

(5 marks)

Q2.(a) Differentiate $y = 3x^2 - 5x$ from first principles.

(7 marks)

(b) Differentiate each of the following:

(i) $y = \sqrt{\cos(2x + 3)}$

(ii) $s = e^{-3t}(t^2 - 4)$

(iii) $y = \frac{\ln(2x - 5)}{(5x - 3)^2}$

(16 marks)

(c) A closed tank with a square base of side x cm is made from a thin sheet of metal.

The total surface area of the tank is 30cm^2 . Show that the volume V can be

expressed as $V = \frac{15x - x^3}{2}$.

Determine the dimensions of the tank so that the volume is maximum.

Find the maximum volume.

(12 marks)

Q.3(a) Determine each of the following integrals:

(i) $\int_2^4 \frac{x^3 - 5 + 3x}{x^2} dx$

(ii) $\int_1^2 \frac{dx}{\sqrt{7 - 2x}}$

(iii) $\int_1^2 (2 - 4x).e^{x-x^2} dx$

(21 marks)

(b) Show that the curve $y = x^3 - 2x^2 - 15x$ crosses the x -axis at $x = 0$, $x = -3$ and at $x = 5$. Find the area bounded by the function and the x -axis. Sketch the function showing the bounded area.

(14 marks)

- Q.4 The equilibrium constant for a chemical reaction was measured by a group of sixty students. The results are presented in the table below:

Equilibrium Constant, K	No. of Students
10.0 but less than 11.0	2
11.0 but less than 12.0	3
12.0 but less than 12.5	8
12.5 but less than 13.0	15
13.0 but less than 13.5	20
13.5 but less than 14.0	10
14.0 but less than 15.5	2

- (a) Calculate the mean percentage yield (\bar{x}) and the standard deviation (s) from the mean.
(16 marks)
- (b) Represent the data on a histogram. Use your graph to estimate
(i) the percentage of results falling in the range $(\bar{x} - s)$ to $(\bar{x} + s)$
(ii) the median of the data.
(16 marks)
- (c) What do you understand by a skewed distribution? Comment on the shape of the distribution of data above.
(3 marks)

Standard Results of Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$	
x^n	nx^{n-1}	
$\ln x$	$\frac{1}{x}$	
e^x	e^x	
e^{ax}	ae^{ax} $a = \text{constant}$	
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
u, v	$u \frac{dv}{dx} + v \frac{du}{dx}$... Product Rule
$\frac{u}{v}$	$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$... Quotient Rule

Standard Integrals

$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} \quad n \neq -1$
$\frac{1}{x}$	$\ln x$
e^x	e^x
e^{ax}	$\frac{1}{a} e^{ax} \quad a = \text{constant}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$

Statistical Formulae:

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation } s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

Assumed Mean Method:

$$\text{Mean } (\bar{x}) = a + c \left(\frac{\sum f(d/c)}{\sum f} \right)$$

$$\text{Standard Deviation } s = c \sqrt{\frac{\sum f(d/c)^2}{\sum f} - \left(\frac{\sum f(d/c)}{\sum f} \right)^2}$$