

**CORK INSTITUTE OF TECHNOLOGY  
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ**

**Semester 2 Examinations 2009/10**

**Module Title: Calculus & Statistics**

**Module Code:** MATH 6002

**School:** School of Science

**Programme Title:**

B.Sc in Applied Biosciences – Year 1

B.Sc in Analytical & Pharmaceutical Chemistry – Year1

B.Sc in Analytical Chemistry & Quality Assurance – Year1

B.Sc in Nutrition & Health Science – Year 1

B.Sc in Pharmaceutical Biotechnology – Year 1

**Programme Code:**

SBIOS\_7\_Y1

SCHEM\_7\_Y1

SCHQA\_8\_Y1

SHNSC\_8\_Y1

SPHB\_8\_Y\_1

**External Examiner(s):** Dr. P. Kirwan

**Internal Examiner(s):** Ms H. Lordan, Ms. F. Wood.

**Instructions:** Answer QUESTION 1 (compulsory - 30 marks)  
and TWO other questions (35 marks each)

**Duration:** 2 Hours

**Sitting:** Autumn 2010

**Requirements for this examination:** Mathematical Tables

**Note to Candidates:** Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.

Q1. Answer **each** of the following:

- (a) A body moves in a straight line in such a way that its displacement,  $s$  metres from its starting point, is given by  $s = 6t^2 - 2t + 3$  where  $t$  is time in seconds. Find

(i) the velocity when  $t$  is 7 seconds

(ii) the acceleration at this point. (5 marks)

- (b) The mass  $M$  of a radioactive material in grams present at any instant  $t$  seconds is given by  $M = 150e^{-0.0012t}$ .

Find

(i) the mass present after 30 seconds.

(ii) the rate of change of mass after one hour.

(5 marks)

- (c) Solve the differential equation  $\frac{dy}{dx} = -3\sin(x)$ , given that  $y = 3$  when  $x = 0$ .

Sketch the solution over the interval  $[0, 2\pi]$ . (5 marks)

- (d) Evaluate  $\int_1^3 (2x + 3)(x - 1) dx$  (5 marks)

- (e) Determine the mean, median and mode of  
2, 3, 7, 5, 5, 13, 1, 7, 4, 8, 3, 4, 3.

(5 marks)

- (f) The concentration of potassium bromate in four samples of wheat flour was recorded as 22.4, 23.2, 25.1 and 25.4 mg/kg. Calculate the standard deviation of the sample data.

(5 marks)

Q2. (a) Differentiate  $f(x) = x^2 + 5x - 6$  from first principles.

(7 marks)

(b) Differentiate each of the following by rule:

(i)  $y = 3x^4 - \frac{4}{3x^2} - 7\sqrt{x} + 15x$  (4 marks)

(ii)  $y = x^3 \ln x$  (6 marks)

(iii)  $y = \frac{x^2 + 2x}{2x + 3}$  (6 marks)

(c) A piece of wire 60m long is bent to form a rectangle. Show, using differentiation that the enclosed area is maximized when the dimensions are that of a square.

(12 marks)

Q3.(a) The slope of the tangent to a curve is given by  $\frac{dy}{dx} = x^3 + 2$ .

Find the equation of the curve if  $y = 11$  when  $x = 2$ .

(5 marks)

(b) Determine each of the following integrals:

(i)  $\int \left( 3x^4 - \frac{2}{7x^3} + \frac{4}{e^x} - \frac{5}{x} \right) dx$  (5 marks)

(ii)  $\int_1^3 5 \cos(3 - 2x) dx$  (6 marks)

(iii)  $\int_{2.0}^{2.5} 8x.e^{5-x^2} dx$  (7 marks)

(c) A curve  $y_1 = 3x^2 - 4x + 10$  is crossed by the line  $y_2 = 40 - 13x$  as shown in Fig.1

(i) Show by solution that  $y_1$  and  $y_2$  intersect at  $x = -5$  and at  $x = 2$ . (6 marks)

(ii) Hence find the area bounded by the two graphs. (6 marks)

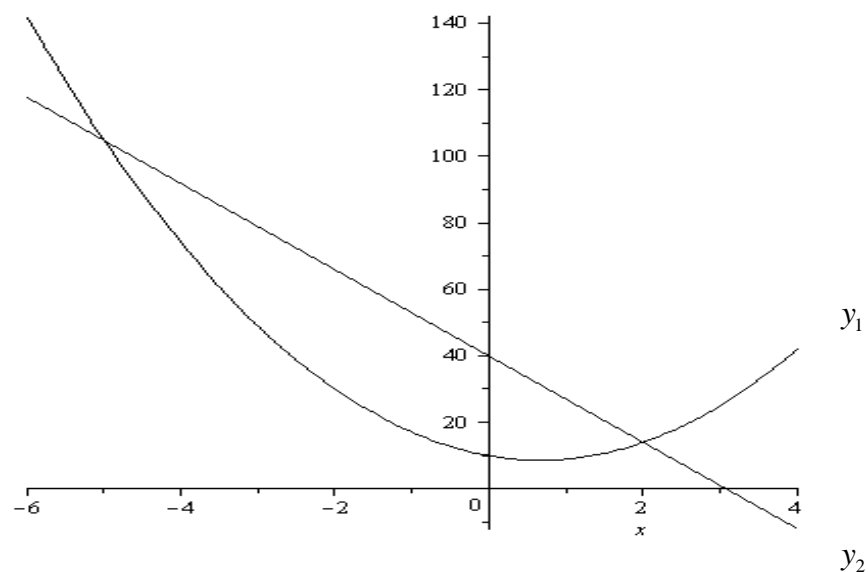


Fig.1

Q4.(a) In experiments involving the precipitation of barium sulphate from a solution of barium chloride the following yields in grammes were recorded:

Yield (g)	No. of Experiments
at least 8.0 but less than 8.1	4
at least 8.1 but less than 8.2	14
at least 8.2 but less than 8.4	26
at least 8.4 but less than 8.6	21
at least 8.6 but less than 8.8	16
at least 8.8 but less than 9.1	3
at least 9.1 but less than 9.4	6

(i) Calculate the mean yield of barium sulphate ( $\bar{x}$ ) and the standard deviation ( $s$ ) from the mean.

(14 marks)

(ii) Represent the distribution on a histogram.

(9 marks)

(iii) Use your graph to estimate the mode of the data.

(2 marks)

(iv) Use your graph to estimate the percentage of data lying in the range  $(\bar{x} - s)$  to  $(\bar{x} + s)$ .

(5 marks)

(b) Explain the terms : (i) quartile values  
(ii) skewed distribution

(5 marks)

# Standard Results of Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$	
$x^n$	$nx^{n-1}$	
$\ln x$	$\frac{1}{x}$	
$e^x$	$e^x$	
$e^{ax}$	$ae^{ax}$	$a = \text{constant}$
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
$u.v$	$u \frac{dv}{dx} + v \frac{du}{dx}$	... Product Rule
$\frac{u}{v}$	$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	... Quotient Rule

### Standard Integrals

$f(x)$	$\int f(x)dx$
$x^n$	$\frac{x^{n+1}}{n+1} \quad n \neq -1$
$\frac{1}{x}$	$\ln x$
$e^x$	$e^x$
$e^{ax}$	$\frac{1}{a}e^{ax} \quad a = \text{constant}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$

### Statistical Formulae:

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation } s = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

