CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2010/11

Module Title: Biostatistics & Data Analysis (CA)

Module Code: MATH7001

School: Biological Sciences

Programme Title: Bachelor of Science (Honours) in Herbal Science – Stage 2

Programme Code: SHERB_8_Y2

External Examiner(s): Dr. J. Reilly
Internal Examiner(s): Dr. T. Creedon

Instructions: Answer FOUR questions. All questions carry equal marks.

Duration: 2 hours

Sitting: Autumn 2011

Requirements for this examination: Log Tables (old version). NB – Forumlae for Statistics and Probability and Statistical Tables are provided at the back of this exam booklet.

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

Q1. In a trial on fifty patients it was found that the times, to the nearest minute, taken for the effect of a drug to wear off were as given below:

128	119	95	107	124	128	147	98	108	117
113	109	124	111	107	139	136	123	112	132
146	128	103	125	114	109	100	131	113	111
124	131	133	131	88	118	116	98	113	127
104	111	112	104	152	117	122	117	95	117

(a) Construct a grouped frequency distribution of the above data.
Groups are (≥ 80 and < 90), (≥ 90 and < 100), etc. (5 marks)

(b) Hence calculate the mean and the standard deviation of the data. (5 marks)

(c) Construct a cumulative frequency curve (an ogive) of the data. (5 marks)

(d) Estimate the median and the first and third quartiles using the ogive. (5 marks)

(e) Construct a Boxplot of the above data. Do you detect any outliers? (5 marks)

Q2. (a) Two drugs (Drug A and Drug B) are used in the treatment of a large group of cancer patients. Of those patients treated with Drug A, 5% experience side effects; of those treated with Drug B, 8% experience side effects. Drug A is used to treat 70% of patients with Drug B used to treat the rest. Calculate the probability that a patient picked at random is

(i) treated with Drug A and experiencing side effects;
(ii) treated with by machine B and does not experiencing side effects;
(iii) experiencing side effects.

(9 marks)

(b) Hospital records show that 10% of the cases of a certain disease are fatal. If 5 patients suffer from this disease, determine the probability that

(i) all will recover
(ii) at least 3 will recover
(iii) exactly 3 will die

(Use the Binomial distribution.) (8 marks)

(c) The incidence of a certain type of cancer in a given region is on average 25 cases per year.
(i) What is the probability that there will be exactly 13 cases in a 6 month period?
(ii) What is the probability that there will be exactly 30 cases in a two-year period?

(Use the Poisson distribution.) (8 marks)

Q3. Women's body temperatures are normally distributed with mean temperature of $37\ ^0C$ and a standard deviation of 0.4^0C.

(a) What percentage of women have body temperatures between $36\ ^0C$ and $38\ ^0C$?

(7 marks)

(b) Between what two values do the middle 90 % of women's body temperatures lie?

(9 marks)

(c) A random sample of 40 women's body temperatures was selected. Find the probability that the mean body temperature of the sample is less than $36.9\ ^0C$.

(9 marks)

Q4. (a) A random sample of 12 people working for a particular company have their heart rates measured with the following results:

61	63	65	70	73	64	75	71	64	65	60	69

(i) Calculate the sample mean and the sample standard deviation.

(ii) Find a 95 % confidence interval for the mean heart rate.

(8 marks)

(b) A random sample of 60 workers was taken at a large lead manufacturing plant and the mean grey matter volume for these workers was found to be 589 cm^3 with a standard deviation of 8 cm^3.

(i) Calculate a 95 % confidence interval for the mean grey matter volume of all workers.

(ii) What sample size would be necessary to estimate the population mean to within 2 cm^3 with 99 % confidence?

(9 marks)

(c) In a study of immunotherapy for children with asthma, out of 50 patients sampled, 37 showed an improvement in breathing.

(i) Calculate a 99 % confidence interval for the population proportion of patients who experience improved breathing due to immunotherapy.

(ii) In estimating the proportion of asthma patients who benefit from immunotherapy, how large should a survey of patients be if you want the resulting sample proportion to differ from the true population proportion by less than 0.01 with 98 % certainty?

(8 marks)

Q5. (a) A respiratory disturbance index (RDI) of more than 30 events per hour is considered evidence of severe sleep disordered breathing (SDB).
A random sample of 100 people with SDB was taken and the sample mean RDI was found to be 32 events per hour with a sample standard deviation of 10 events per hour.
Is there evidence that the mean RDI is greater than 30 events per hour?
(Use a 0.05 level of significance)

(7 marks)

(b) A hospital spokesperson that not more than 5 % of patients are dissatisfied. In a sample of 40 patients, 4 are found to be dissatisfied. Test if we can reject the spokesperson's claim based on this sample. Conduct the test at a 0.01 level of significance. (8 marks)

(c) Two groups of 10 patients each were given two different soporific drugs. The increase in hours sleep for each patient is shown in the table below:

Patient	1	2	3	4	5	6	7	8	9	10
Drug A	0.7	-1.6	-0.2	-1.2	-0.1	3.4	3.7	0.8	0.0	2.0
Drug B	1.9	0.8	1.1	0.1	-0.1	4.4	5.5	1.6	4.6	3.4

Test the hypothesis that patients given Drug B have a higher increase in sleep hours than patients given Drug A. (Use a 0.01 level of significance.)

(10 marks)

Q6. (a) Data on the number of sick days taken by two categories of worker are shown in the table below.

Category of worker	Sick Days		
	None	One	More than One
Industrial	139	212	149
Clerical	66	94	40

Do these figures suggest a relationship between the category of worker and the number of sick days taken? Test at a 0.05 level of significance using the χ^2 distribution.

(12 marks)

(b) A recent dentist's project was looking at the affect of smoking on the health of oral implants in a population of elderly patients. The table below shows the number of healthy and unhealthy implants for smokers and non-smokers. Does smoking affect the health of the implant? Test at the 0.01 level of significance.

	Healthy Implant	
	Yes	No
Smoker	32	48
Non-smoker	42	7

(13 marks)

Q7. If a person dives into cold water, a neural reflex response automatically shuts off blood circulation to the skin and muscles and reduces the pulse rate. A medical research team conducted an experiment using a group of five two-year olds. A child's face was placed momentarily in cold water, and the corresponding reduction in pulse rate was recorded. The data are given in the following table.

Water Temperature, in ${}^0C$, (x)	10	13	15	18	21
Pulse rate reduction (y)	15	13	10	6	2

(a) Plot the data on a scatter diagram.

(3 marks)

(b) Calculate the regression line and fit this line to the scatter diagram.

(8 marks)

(c) Calculate the correlation coefficient and interpret your answer.

(7 marks)

(d) Evaluate the coefficient of determination and interpret your answer.

(4 marks)

(e) If a child's face is placed in water of temperature is $20\,{}^0C$, estimate the reduction in pulse rate.

(3 marks)

Formulae

Descriptive Statistics

Mean : $\bar{x} = \frac{\sum x}{n}$

Mean of a frequency distribution: $\bar{x} = \frac{\sum fx}{\sum f}$

Population standard deviation: $\sqrt{\frac{\sum (x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$

Population standard deviation: $\sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$

Sample standard deviation = $s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$

Sample standard deviation = $s = \sqrt{\frac{\sum fx^2 - \frac{(\sum fx)^2}{\sum f}}{(\sum f)-1}}$

Mode = $L + \left(\frac{D_1}{D_1 + D_2}\right)C$

Skewness = $\frac{\text{mean} - \text{mode}}{\text{s.d.}} = \frac{3(\text{mean} - \text{median})}{\text{s.d.}}$

coefficient of variation = $\frac{s}{\bar{x}} \times 100\%$

Probability

Addition Law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Law $P(A \cap B) = P(A|B)P(B)$

Bayes' Theorem: $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$

Binomial $P(X = r) = {}^nC_r p^r q^{n-r}$

Poisson $P(X = r) = \frac{e^{-m} m^r}{r!}$

Central Limit Formulae

X is $N(\mu,\sigma) \Rightarrow Z = \dfrac{X-\mu}{\sigma}$ and $\overline{X}$ is $N(\mu,\dfrac{\sigma}{\sqrt{n}})$

Confidence Interval for the mean μ:

$$\overline{X} \pm Z\frac{\sigma}{\sqrt{n}} \quad \text{or} \quad \overline{X} \pm Z\frac{s}{\sqrt{n}} \quad \text{or} \quad \overline{X} \pm t_{n-1}\frac{s}{\sqrt{n}}$$

Confidence interval for the variance σ^2:

A $100(1-\alpha)$ % confidence interval for the variance is $\dfrac{(n-1)s^2}{\chi^2_{hi}} < \sigma^2 < \dfrac{(n-1)s^2}{\chi^2_{lo}}$

where $\chi^2_{hi} = \chi^2_{\frac{\alpha}{2},n-1}$ and $\chi^2_{lo} = \chi^2_{1-\frac{\alpha}{2},n-1}$

Confidence Interval for the proportion P:

$$p_s \pm Z\sqrt{\frac{p_s(1-p_s)}{n}}$$

Sample size calculation for the mean: $n = \dfrac{Z^2 s^2}{E^2}$

Sample size calculation for the proportion: $n = \dfrac{Z^2 p_s(1-p_s)}{E^2}$

One sample Hypothesis Testing for the mean:

$$Z = \frac{\overline{X}-\mu}{\frac{\sigma}{\sqrt{n}}} \quad \text{or} \quad Z = \frac{\overline{X}-\mu}{\frac{s}{\sqrt{n}}} \quad \text{or} \quad t = \frac{\overline{X}-\mu}{\frac{s}{\sqrt{n}}}$$

Two sample Hypothesis Testing for comparing the means:

$$Z = \frac{\overline{X_1} - \overline{X_2} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

or

$$t = \frac{\overline{X_1} - \overline{X_2} - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

Hypothesis testing for the proportion:

$$Z = \frac{p_s - P}{\sqrt{\frac{p_s(1 - p_s)}{n}}}$$

Chi-Squared Distribution

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

Regression and Correlation

Least Squares Regression Line: $y = ax + b$

$$a = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b = \frac{\sum y - a\sum x}{n}$$

Coefficient of Correlation:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

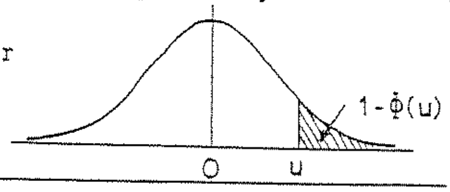
Coefficient of determination:

cd = r^2

Table 3

AREAS IN TAIL OF THE NORMAL DISTRIBUTION

The function tabulated is $1 - \Phi(u)$ where $\Phi(u)$ is the cumulative distribution function of a standardised Normal variable u. Thus $1 - \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-u^2/2}\,du$ is the probability that a standardised Normal variable selected at random will be greater than a value of u $\left(= \frac{x-\mu}{\sigma}\right)$

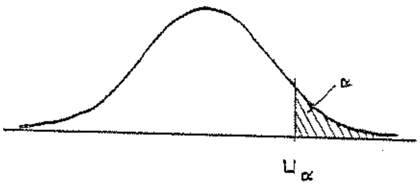


$\frac{(x-\mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									
3.1	.00097									
3.2	.00069									
3.3	.00048									
3.4	.00034									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3.8	.00007									
3.9	.00005									
4.0	.00003									

Table 4

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

The table gives the 100α percentage points, u_α, of a standardised Normal distribution where $\alpha = \frac{1}{\sqrt{2\pi}} \int_{u_\alpha}^{\infty} e^{-u^2/2}\, du$. Thus u_α is the value of a standardised Normal variate which has probability α of being exceeded.



α	u_α	α	u_α	α	u_α	α	u_α	α	u_α	α	u_α
.50	0.0000	.050	1.6449	.030	1.8808	.020	2.0537	.010	2.3263	.050	1.6449
.45	0.1257	.048	1.6646	.029	1.8957	.019	2.0749	.009	2.3656	.010	2.3263
.40	0.2533	.046	1.6849	.028	1.9110	.018	2.0969	.008	2.4089	.001	3.0902
.35	0.3853	.044	1.7060	.027	1.9268	.017	2.1201	.007	2.4573	.0001	3.7190
.30	0.5244	.042	1.7279	.026	1.9431	.016	2.1444	.006	2.5121	.00001	4.2649
.25	0.6745	.040	1.7507	.025	1.9600	.015	2.1701	.005	2.5758	.025	1.9600
.20	0.8416	.038	1.7744	.024	1.9774	.014	2.1973	.004	2.6521	.005	2.5758
.15	1.0364	.036	1.7991	.023	1.9954	.013	2.2262	.003	2.7478	.0005	3.2905
.10	1.2816	.034	1.8250	.022	2.0141	.012	2.2571	.002	2.8782	.00005	3.8906
.05	1.6449	.032	1.8522	.021	2.0335	.011	2.2904	.001	3.0902	.000005	4.4172

Table 5

ORDINATES OF THE NORMAL DISTRIBUTION

The table gives ϕ (u) for values of the standardised Normal variate, u, in the interval 0.0(0.1)4.0 where $\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}$

u	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.0	.3989	.3970	.3910	.3814	.3683	.3521	.3332	.3123	.2897	.2661
1.0	.2420	.2179	.1942	.1714	.1497	.1295	.1109	.0940	.0790	.0656
2.0	.0540	.0440	.0355	.0283	.0224	.0175	.0136	.0104	.0079	.0060
3.0	.0044	.0033	.0024	.0017	.0012	.0009	.0006	.0004	.0003	.0002
4.0	.0001									

Table 7

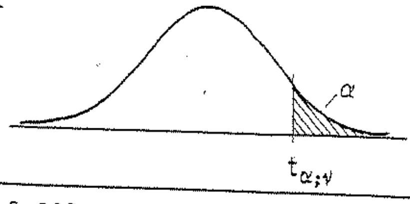
PERCENTAGE POINTS OF THE t DISTRIBUTION

The table gives the value of $t_{\alpha;\nu}$ — the 100α percentage point of the t distribution for ν degrees of freedom.

The values of t are obtained by solution of the equation:-

$$\alpha = \Gamma\{\tfrac{1}{2}(\nu+1)\}\{\Gamma(\tfrac{1}{2}\nu)\}^{-1}(\nu\pi)^{-1/2}\int_t^{\infty}(1+x^2/\nu)^{-(\nu+1)/2}dx$$

Note. The tabulation is for one tail only i.e. for positive values of t. For |t| the column headings for α must be doubled.



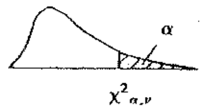
$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$\nu =$ 1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

This table is taken from Table III of Fisher & Yates: Statistical Tables for Biological, Agricultural and Medical Research, published by Oliver & Boyd Ltd., Edinburgh, and by permission of the authors and publishers and also from Table 12 of Biometrika Tables for Statisticians, Volume 1, by permission of the Biometrika Trustees.

Table 8

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

Table of $\chi^2_{\alpha;\nu}$ — the 100 α percentage point of the χ^2 distribution for ν degrees of freedom



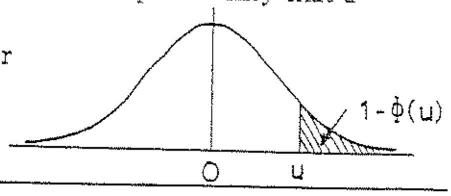
α =	.995	.99	.98	.975	.95	.90	.80	.75	.70	.50	.30	.25	.20	.10	.05	.025	.02	.01	.005	.001	= α
ν = 1	$.0^4393$	$.0^3157$	$.0^3628$	$.0^3982$	.00393	.0158	.0642	.102	.148	.455	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	ν = 1
2	.0100	.0201	.0404	.0506	.103	.211	.446	.575	.713	1.386	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2
3	.0717	.115	.185	.216	.352	.584	1.005	1.213	1.424	2.366	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3
4	.207	.297	.429	.484	.711	1.064	1.649	1.923	2.195	3.357	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4
5	.412	.554	.752	.831	1.145	1.610	2.343	2.675	3.000	4.351	6.064	6.626	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517	5
6	.676	.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6
7	.989	1.239	1.564	1.690	2.167	2.833	3.822	4.255	4.671	6.346	8.383	9.037	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	16.222	17.117	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.036	11.721	14.339	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	18.418	19.369	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	21.689	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	24.939	26.039	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	18.137	19.021	22.337	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.867	24.337	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336	30.319	31.528	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.336	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	33.660	34.872	39.335	44.165	45.616	47.269	51.805	55.759	59.342	60.436	63.691	66.766	73.402	40
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.661	50
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	65.227	66.981	68.972	74.397	79.082	83.298	84.580	88.379	91.952	99.607	60
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	61.698	63.346	69.334	75.689	77.577	79.715	85.527	90.531	95.023	96.388	100.425	104.215	112.317	70
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	71.145	72.915	79.334	86.120	88.130	90.405	96.578	101.880	106.629	108.069	112.329	116.321	124.839	80
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	80.625	82.511	89.334	96.524	98.650	101.054	107.565	113.145	118.136	119.648	124.116	128.299	137.208	90
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	90.133	92.129	99.334	106.906	109.141	111.667	118.498	124.342	129.561	131.142	135.807	140.170	149.449	100

For values of $\nu > 30$, approximate values for χ^2 may be obtained from the expression $\nu\left[1 - \frac{2}{9\nu} \pm \frac{X}{\sigma}\sqrt{\frac{2}{9\nu}}\right]^3$, where $\frac{X}{\sigma}$ is the normal deviate cutting off the corresponding tails of a normal distribution. If $\frac{X}{\sigma}$ is taken at the 0.02 level, so that 0.01 of the normal distribution is in each tail, the expression yields χ^2 at the 0.99 and 0.01 points. For very large values of ν it is sufficiently accurate to compute $\sqrt{2\chi^2}$, the distribution of which is approximately normal around a mean of $\sqrt{2\nu - 1}$ and with a standard deviation of 1. This table is taken by consent from Statistical Tables for Biological, Agricultural, and Medical Research, by R. A. Fisher and F. Yates, published by Oliver and Boyd, Edinburgh, and from Table 8 of Biometrika Tables for Statisticians, Vol. 1, by permission of the Biometrika Trustees.

Table 3

AREAS IN TAIL OF THE NORMAL DISTRIBUTION

The function tabulated is $1 - \Phi(u)$ where $\Phi(u)$ is the cumulative distribution function of a standardised Normal variable u. Thus $1 - \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-u^2/2}\, du$ is the probability that a standardised Normal variable selected at random will be greater than a value of u $\left(= \frac{x-\mu}{\sigma}\right)$



$\frac{(x-\mu)}{\sigma}$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									
3.1	.00097									
3.2	.00069									
3.3	.00048									
3.4	.00034									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3.8	.00007									
3.9	.00005									
4.0	.00003									

Scan all

Table 7

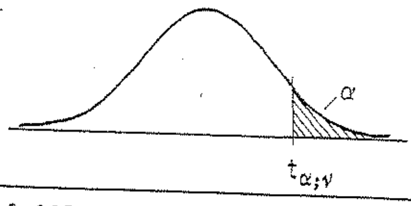
PERCENTAGE POINTS OF THE t DISTRIBUTION

The table gives the value of $t_{\alpha;\nu}$ — the 100α percentage point of the t distribution for ν degrees of freedom.

The values of t are obtained by solution of the equation:-

$$\alpha = \Gamma\{\tfrac{1}{2}(\nu+1)\}\,\{\Gamma(\tfrac{1}{2}\nu)\}^{-1}\,(\nu\pi)^{-1/2}\int_t^{\infty}(1 + x^2/\nu)^{-(\nu+1)/2}dx$$

Note. The tabulation is for one tail only i.e. for positive values of t. For |t| the column headings for α must be doubled.



$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
ν = 1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

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