

**CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ**

Autumn Examinations 2010/11

Module Title: Calculus & Statistics

Module Code: MATH 6002

School: School of Science

Programme Title(s):

B.Sc in Applied Biosciences – Year 1

B.Sc in Analytical & Pharmaceutical Chemistry – Year1

B.Sc in Analytical Chemistry & Quality Assurance – Year1

B.Sc in Nutrition & Health Science – Year 1

B.Sc in Pharmaceutical Biotechnology – Year 1

Programme Code(s):

SBIOS_7_Y1

SCHEM_7_Y1

SCHQA_8_Y1

SHNSC_8_Y1

SPHB_8_Y_1

External Examiner(s): Dr. P. Kirwan

Internal Examiner(s): Ms H. Lordan, Ms. F. Wood.

Instructions: Answer QUESTION 1 (compulsory - 30 marks)
and TWO other questions (35 marks each)

Duration: 2 Hours

Sitting: Autumn 2011

Requirements for this examination: Mathematical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

Q1. Answer **each** of the following:

- (a) Show that $y = e^{2x}$ satisfies the equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$.

Show that $y = e^{-x}$ is also a solution.

(5 marks)

- (b) The heat capacity C of a gas varies with absolute temperature θ as shown:

$$C = 21 + 0.72\theta - 1.2 \times 10^{-3}\theta^2$$

Determine the maximum value of C and the temperature at which it occurs.

(5 marks)

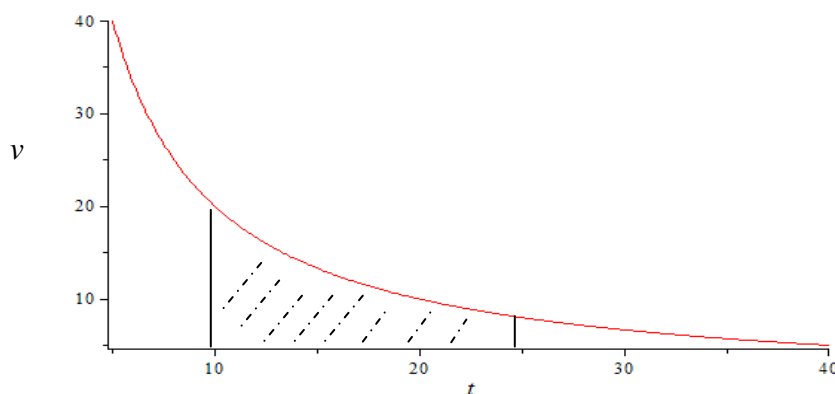
- (c) Evaluate $\int_1^3 (2x-1)^2 dx$

(5 marks)

- (d) The velocity of an object is described by the equation $v = \frac{200}{t}$ where v is the velocity in ms^{-1} and t is time in seconds.

The area under the velocity-time graph represents the distance travelled by the object.

Calculate the distance travelled in the interval from $t = 10$ to $t = 25$ s.



$$\text{Area} = \int y \cdot dx$$

(5 marks)

cont'd/...

- (e) The average age of a group of 6 people at a party was 20. The average age increased by 2 years when three new people joined the group. What is the average age of the three new people who joined the group?

(5 marks)

- (f) A distribution of data is given as follows:

variable: x	3	4	6	7	8
frequency: f	1	2	1	3	2

Find

(i) the mean mark (\bar{x})

(ii) the median

Show that $\sum (x - \bar{x}) = 0$.

(5 marks)

Q2.(a) Differentiate each of the following :

(i) $y = 5x^2 + \frac{2}{x} - \sqrt{x} + 5$

(ii) $y = e^{-3x} \cos(2x)$

(iii) $y = \frac{\ln(1+x^2)}{(1-x)^3}$

(16 marks)

- (b) Find three turning points on the curve $y = x^4 - 8x^2$. Identify the nature of each and hence sketch the curve.

(12 marks)

- (c) The distance travelled by the object is described by the equation

$$s = 8 + \frac{4}{t^2}$$

where t is the time in seconds and s is in metres.

- (i) Copy and complete the table below and hence sketch the relationship.

t	0.5	1	2	3
s				

- (ii) Find the velocity $\left(\frac{ds}{dt}\right)$ of the object at $t = 1.5$ s.

(7 marks)

Q3.(a) Determine each of the following:

(i) $\int_1^3 (5x - 2)^4 dx$

(ii) $\int \sqrt{3x^2 - 5x + 4} (6x - 5) dx$

(iii) $\int_0^1 (\sin x + \cos 2x) dx$

(21 marks)

- (b) Find the points of intersection of the curve $y = x^2 + 2$ and the line $y = 14 - x$. Sketch the curve and the line. Find the area enclosed by the curve and the line.

(14 marks)

- Q4. The marks awarded to 120 students in an examination were recorded to the nearest percentage and tabulated as follows:

Mark Awarded (%)	Number of students
At least 24 but less than 30	5
At least 30 but less than 36	26
At least 36 but less than 42	40
At least 42 but less than 48	22
At least 48 but less than 54	17
At least 54 but less than 66	8
At least 66 but less than 90	2

- (a) Taking the mid-interval value of each class, calculate the mean (\bar{x}) and the standard deviation (s) from the mean.
(16 marks)
- (b) Represent this information on a cumulative frequency table. Hence draw the cumulative frequency polygon (ogive).
(13 marks)
- (c) Use the ogive to estimate
- (i) the median mark
 - (ii) the lower quartile and upper quartile values
 - (iii) the inter-quartile range.

(6 marks)

Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$

Product Rule

$$y = uv$$

$$\Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

Quotient Rule

$$y = \frac{u}{v}$$

$$\Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Chain Rule

$$f(x) = u(v(x))$$

$$\Rightarrow f'(x) = \frac{du}{dv} \frac{dv}{dx}$$

Standard Integrals

$f(x)$	$\int f(x) dx$
x^n	$\frac{x^{n+1}}{n+1} \quad n \neq -1$
$\frac{1}{x}$	$\ln x$
e^x	e^x
e^{ax}	$\frac{1}{a} e^{ax}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$

Statistical Formulae:

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation } s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

Assumed Mean Method:

$$\text{Mean } (\bar{x}) = a + c \left(\frac{\sum f(d/c)}{\sum f} \right)$$

$$\text{Standard Deviation } s = c \sqrt{\frac{\sum f(d/c)^2}{\sum f} - \left(\frac{\sum f(d/c)}{\sum f} \right)^2}$$