

CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2010/11

Module Title: Calculus & Statistics

Module Code: MATH 6002

School: School of Science

Programme Title:

B.Sc in Applied Biosciences – Year 1

B.Sc in Analytical & Pharmaceutical Chemistry – Year 1

B.Sc in Analytical Chemistry & Quality Assurance – Year 1

B.Sc in Nutrition & Health Science – Year 1

B.Sc in Pharmaceutical Biotechnology – Year 1

Programme Code:

SBIOS_7_Y1

SCHEM_7_Y1

SCHQA_8_Y1

SHNSC_8_Y1

SPHB_8_Y_1

External Examiner(s): Dr. P. Kirwan

Internal Examiner(s): Ms H. Lordan, Ms. F. Wood.

Instructions: Answer QUESTION 1 (compulsory - 30 marks)
and TWO other questions (35 marks each)

Duration: 2 Hours

Sitting: Summer 2011

Requirements for this examination: Mathematical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.

Q1. Answer **each** of the following:

(a) Differentiate each of the following:

(i) $y = \frac{x^4}{3} - \frac{3}{x^4}$

(ii) $F = s^2 + 2\sqrt{s} - 15$ (5 marks)

(b) An object is thrown vertically upwards. The height travelled by the object is described by the equation

$$h = 20t - 4.9t^2$$

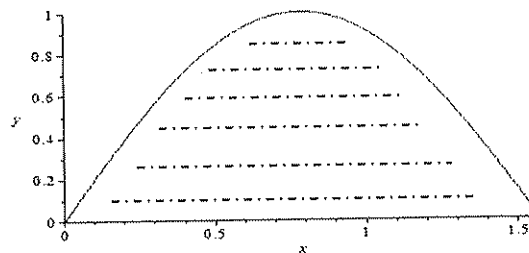
where t is the time in seconds and h is in metres.

(i) How long does it take for the object to reach its maximum height?

(ii) What is the maximum height attained? (5 marks)

(c) Evaluate $\int_1^2 (x-3)(2x+5) dx$ (5 marks)

(d) A graph of $y = \sin(2x)$ is shown in the interval $x = 0$ to $x = \frac{\pi}{2}$.
Calculate the shaded area.



(5 marks)

(e) Two sets of data are given as follows:

Set A: 13, 8, 10, 14, 10, 13

Set B: 10, 8, 11, 12, x , 11

(i) Find x such that both data sets have the same mean value.

(ii) Which data set has the greater dispersion? Justify your answer.

(5 marks)

cont'd / ...

- (f) Six students in a class get the following marks out of 20 in an assessment:

Student	A	B	C	D	E	F
Mark	4	8	11	15	16	18

Find the mean mark (\bar{x}) and the standard deviation (s). (5 marks)

Q2.(a) Differentiate each of the following by rule:

(i) $y = 4x^2 \cdot e^{-x^2}$

(ii) $y = \frac{\sin(2x)}{x^2 - 9x}$

(iii) $i = 20 \cos(40\pi t - 0.5)$

(16 marks)

- (b) Find two turning points on the curve $y = x^3 + 6x^2$. Identify the nature of each and hence sketch the curve.

Find the value of x for which a tangent to the curve has a slope of 15.

(13 marks)

- (c) A chemical solution is heated to 60°C and then allowed to cool naturally. The temperature of the cooling solution $\theta (^\circ\text{C})$ at any time t , (mins) is given by

$$\theta = 60e^{-0.67t}$$

Find:

- (i) the temperature of the cooling solution after 3 minutes
(ii) the rate at which the solution is cooling at this instant.

(6 marks)

Q3.(a) Determine each of the following:

(i) $\int \left(\frac{4x^5 - 3 + x}{x^3} \right) dx$

(ii) $\int_0^1 2xe^{3x^2+1} dx$

(iii) $\int_0^2 (4t-3)^6 dt$

(21 marks)

(b) Graph the function $f(x) = x^2 - 2x - 8$ from $x = -2$ to $x = 6$.

Find the area bounded by the curve, the x -axis and the ordinates $x = -2$ and $x = 6$.

(14 marks)

- Q4. The cyanide concentration (milligrams per kilogram of soil) was measured and the results were tabulated as follows:

Concentration (mg/kg)	Number of Samples
At least 62.0 but less than 62.4	2
At least 62.4 but less than 62.8	7
At least 62.8 but less than 63.2	10
At least 63.2 but less than 63.6	14
At least 63.6 but less than 64.4	12
At least 64.4 but less than 65.2	10
At least 65.2 but less than 66.0	5

- (i) Represent this information on a histogram. Use the histogram to estimate the mode and the median.

(13 marks)

- (ii) Calculate the mean mass of cyanide (\bar{x}) per sample and the standard deviation (s) from the mean.

(16 marks)

- (iii) From the histogram estimate the number of samples falling in the range $(\bar{x} - s)$ to $(\bar{x} + s)$.

(6 marks)

Standard Results of Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$	
x^n	nx^{n-1}	
$\ln x$	$\frac{1}{x}$	
e^x	e^x	
e^{ax}	ae^{ax}	$a = \text{constant}$
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
$u.v$	$u \frac{dv}{dx} + v \frac{du}{dx}$... Product Rule
$\frac{u}{v}$	$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$... Quotient Rule

Standard Integrals

$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} \quad n \neq -1$
$\frac{1}{x}$	$\ln x$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax} \quad a = \text{constant}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$

Statistical Formulae

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation } s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

Assumed Mean Method:

$$\text{Mean } (\bar{x}) = a + c \left(\frac{\sum f(d/c)}{\sum f} \right)$$

$$\text{Standard Deviation } s = c \sqrt{\frac{\sum f(d/c)^2}{\sum f} - \left(\frac{\sum f(d/c)}{\sum f} \right)^2}$$