Cork Institute of Technology

Higher Certificate in Engineering in Electrical Engineering – Award

(National Certificate in Engineering in Electrical Engineering – Award

(NFQ – Level 6)

Autumn 2005

Mathematics

(Time: 3 Hours)

Instructions: Answer ***five*** questions.

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Q1. (a) Determine the stationary values (i.e. the turning points and points of inflection) of the function :

$$f(x) = 2x^3 - 6x^2 - 18x - 9$$

Sketch the graphs of the functions $f(x)$ and $f'(x)$ against x. (10 marks)

(b) Differentiate each of the following with regard to the included variable:

(i) $3\sinh 7x + x\cosh 5x$;

(ii) $\sin^{-1}(2x-5)$ (10 marks)

(c) Use Implicit differentiation to determine the slope of the graph:

$7x^3 - 5y^2 = 3xy^2$, at the point (0,0). (10 marks)

(d) Given that the coordinates of a body moving in a plane, after t seconds are given by

$$x = 5t + \pi\cos^2 t \quad \text{and} \quad y = 2 - \pi\sin t$$

Use parametric differentiation to find $\frac{dy}{dx}$ in terms of t.

Determine the value of $\frac{dy}{dx}$ initially. (10 marks)

Q2. (a) A cable hangs in the catenary $y = c\cosh\frac{x}{c}$ where C is the minimum height and its length for a span of $2a$ is $2L$ where $L = c\sinh\frac{a}{c}$.

Find the span and the height of two equal supports for a cable of length 80 m. where the minimum height is 10m. (18 marks)

(b) The power, p, consumed by a resistor, R, where E is the applied voltage is given by

$$p = \frac{E^2}{R}$$

Using Partial differentiation, determine the percentage change in p when E increases by 4% and R decreases by 2%. (10 marks)

(c) Use the Newton-Raphson method to determine the root of the equation $6\ln x = 20 - 0.5x$ for x correct to two decimal places. Use $x_1 = 8$ as you first estimate. (12 marks)

Q3. (a) Evaluate four of the following integrals:

(i) $\int\left(5x^4 - 7\cos 2x + 3e^{-5x} - \frac{4}{x}\right)dx$

(ii) $\int \sin^4(2x).\cos(2x)dx$ ……..by substitution

(iii) $\int \frac{2x-9}{(2x+5)(x-1)}dx$ ……using partial fractions.

(iv) $\int_0^{0.2} xe^{-2x}dx$ ……by parts

(v) $\int \frac{5}{\sqrt{16-25x^2}}dx$ (4 x 8 marks)

(b) Determine the mean value of the function $y = 20\sin 10\pi t$ between t=0 and t=0.1

(8 marks)

Q4. (a) By means of separating the variables find a general solution for the differential equation

$$x(4y-1)\frac{dy}{dx} = \left(6x^2 - 8x\right)$$

Find also a particular solution if $y = 1.0$ when $x = 0$. (20 marks)

(b) According to Newton's law of cooling the temperature (T) of a body decreases at a rate proportional to the difference between the temperature of the body and the temperature of the surrounding air (A). Display this information by means of a differential equation.

In air which is maintained at 10 degrees Celsius the temperature of a body decreases from 130 degrees to 70 degrees in 15 minutes.

Find an expression for the temperature after t s. Find also the time taken for the temperature of the body to reach 30 degrees Celsius. (20 marks)

Q5. (a) Find the general solution for the second order differential equation:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 21y = 0$$

Hence find its particular solution given that $y = 0$ and $\frac{dy}{dx} = 4$ when $x = 0$

(20 marks)

(b) Write the following equation as a first order linear differential equation:

$$x\frac{dy}{dx} - 5y = x^2 + 3x$$

Solve the equation to find the general solution.

Determine the particular solution given that $y=5$ when $x=1$. (20 marks)

Q6. (a) Solve the following network simultaneous equations for i_1, i_2 and i_3, using Cramer's rule (Determinants) at least once.

$$\begin{aligned} 3i_1 + 5i_2 + i_3 &= 14 \\ 5i_1 - 2i_2 + 3i_3 &= 9 \\ 2i_1 - i_2 &= 7 \end{aligned}$$

(16 marks)

(b) Given the following matrices A and B, determine, where possible,

(i) **C=A-B,** (ii) **D = AB,** and (iii) **E = BA**

$$\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 5 & -2 \\ -1 & 0 \end{pmatrix} \quad ; \quad \mathbf{B} = \begin{pmatrix} -5 & 2 \\ -1 & 6 \end{pmatrix}$$

Find $\mathbf{B^{-1}}$, i.e. the inverse of the matrix **B.** (8 marks)

(c) Show that $Q = \frac{1}{25}\begin{pmatrix} 2 & 7 & 1 \\ 19 & -21 & -3 \\ 15 & -10 & -5 \end{pmatrix}$ is the inverse of the matrix $P = \begin{pmatrix} 3 & 1 & 0 \\ 2 & -1 & 1 \\ 5 & 5 & -7 \end{pmatrix}$

Hence find the values of *A,B* and *C* which satisfy the equations

$$\begin{aligned} 3A + B &= 7 \\ 2A - B + C &= 12 \\ 5A + 5B - 7C &= -23 \end{aligned}$$

(16 marks)

Q7. A computer engineer finds that the sizes (in Kbytes) of sixty files stored on a disk are as follows.

128	119	105	117	124	158	147	128	108	117
113	109	124	111	137	149	136	123	112	132
146	128	143	135	114	109	100	131	113	111
124	131	133	131	128	118	116	140	113	139
104	111	112	108	152	117	122	117	115	117
142	126	152	137	148	129	148	111	118	137

You are required to:

(i) Arrange the data in a grouped frequency distribution of six groups i.e. (100-119) etc;

(ii) Determine the mean and standard deviation of the sample. You may use the statistical mode functions of your calculator;

(iii) Construct an ogive of the data;

(iv) Estimate the median and the first and third quartiles of the sample;

(v) Determine the lifetime above which the top 10 % of lifetimes lie. (5 x8 marks)

Q8. (a) In a survey of a particular city, it was found that 90% of households have a landline telephone while 40 % have a games console and 55% have a personal computer (i.e. PC).

Given that these events occur independently of each other, demonstrate these events on a probability Venn Diagram.

Hence find the probability that a household, chosen at random will experience have ;

(i) Landline phone or a games console;

(ii) Landline phone and PC;

(iii) Only a games console;

(iv) At least one of these items;

(v) Not more than one of these items. (20 marks)

(b) The weights of passengers carried by a major airline are Normally distributed, with a mean of 84 kg. and a standard deviation of 20 kg.

What is the probability that a passenger, chosen at random will weigh:

(i) Less than 100 kg.

(ii) Between 90 and 120 kg.

(iii) More than 60 kg.

Between what two values do the middle 90% of passenger weights lie? (20 marks)

Statistical Formulae

Mean of an array; $\bar{x}=\frac{\sum x}{n}$

Mean of a frequency distribution $\bar{x} = \frac{\sum fx}{\sum f}$. $\bar{x} = a + \frac{\sum fd}{\sum f}$

Standard Deviation of an array: $\sigma=\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ or $\sigma=\sqrt{\frac{\sum x^2}{n}-\left(\frac{\sum x}{n}\right)^2}$

Standard Deviation of a frequency distribution $\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$ or $=\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$

Standard Nornal Units; $Z=\frac{x-\mu}{\sigma}$

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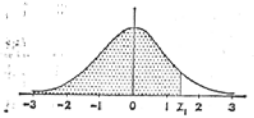
Area under the Normal Curve

$$P(z \leqslant z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2}\, dz .$$

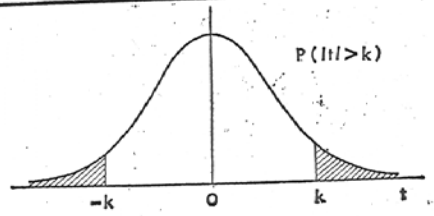


z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0·1	0·5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
0·2	0·5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
0·3	0·6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
0·4	0·6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0·5	0·6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0·6	0·7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0·7	0·7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
0·8	0·7881	7910	7939	7967	7995	8023	8051	8078	8106	8133
0·9	0·8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1·0	0·8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1·1	0·8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1·2	0·8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1·3	0·9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1·4	0·9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1·5	0·9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1·6	0·9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1·7	0·9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1·8	0·9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1·9	0·9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2·0	0·9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2·1	0·9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2·2	0·9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2·3	0·9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2·4	0·9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2·5	0·99379	99396	99413	99430	99446	99461	99477	99492	99506	99520
2·6	0·99534	99547	99560	99573	99585	99598	99609	99621	99632	99643
2·7	0·99653	99664	99674	99683	99693	99702	99711	99720	99728	99736
2·8	0·99744	99752	99760	99767	99774	99781	99788	99795	99801	99807
2·9	0·99813	99819	99825	99831	99836	99841	99846	99851	99856	99861
3·0	0·99865	99869	99874	99878	99882	99886	99889	99893	99897	99900
3·1	0·99903	99906	99910	99913	99916	99918	99921	99924	99926	99929
3·2	0·99931	99934	99936	99938	99940	99942	99944	99946	99948	99950
3·3	0·99952	99953	99955	99957	99958	99960	99961	99962	99964	99965
3·4	0·99966	99968	99969	99970	99971	99972	99973	99974	99975	99976
3·5	0·99977	99978	99978	99979	99980	99981	99981	99982	99983	99983
3·6	0·99984	99985	99985	99986	99986	99987	99987	99988	99988	99989
3·7	0·99989	99990	99990	99990	99991	99991	99992	99992	99992	99992
3·8	0·99993	99993	99993	99994	99994	99994	99994	99995	99995	99995
3·9	0·99995	99995	99996	99996	99996	99996	99996	99996	99997	99997

t-DÁILEADH

t-DISTRIBUTION

ν = n -	20	10	5	2	1	0·2
1	3·078	6·314	12·706	31·821	63·657	318·310
2	1·886	2·920	4·303	6·965	9·925	22·327
3	1·638	2·353	3·182	4·541	5·841	10·215
4	1·533	2·132	2·776	3·747	4·604	7·173
5	1·476	2·015	2·571	3·365	4·032	5·893
6	1·440	1·943	2·447	3·143	3·707	5·208
7	1·415	1·895	2·365	2·998	3·499	4·785
8	1·397	1·860	2·306	2·896	3·355	4·501
9	1·383	1·833	2·262	2·821	3·250	4·297
10	1·372	1·812	2·228	2·764	3·169	4·144
11	1·363	1·796	2·201	2·718	3·106	4·025
12	1·356	1·782	2·179	2·681	3·055	3·930
13	1·350	1·771	2·160	2·650	3·012	3·852
14	1·345	1·761	2·145	2·624	2·977	3·787
15	1·341	1·753	2·131	2·602	2·947	3·733
16	1·337	1·746	2·120	2·583	2·921	3·686
17	1·333	1·740	2·110	2·567	2·898	3·646
18	1·330	1·734	2·101	2·552	2·878	3·610
19	1·328	1·729	2·093	2·539	2·861	3·579
20	1·325	1·725	2·086	2·528	2·845	3·552
21	1·323	1·721	2·080	2·518	2·831	3·527
22	1·321	1·717	2·074	2·508	2·819	3·505
23	1·319	1·714	2·069	2·500	2·807	3·485
24	1·318	1·711	2·064	2·492	2·797	3·467
25	1·316	1·708	2·060	2·485	2·787	3·450
26	1·315	1·706	2·056	2·479	2·779	3·435
27	1·314	1·703	2·052	2·473	2·771	3·421
28	1·313	1·701	2·048	2·467	2·763	3·408
29	1·311	1·699	2·045	2·462	2·756	3·396
30	1·310	1·697	2·042	2·457	2·750	3·385
40	1·303	1·684	2·021	2·423	2·704	3·307
60	1·296	1·671	2·000	2·390	2·660	3·232
120	1·289	1·658	1·980	2·358	2·617	3·160
∞	1·282	1·645	1·960	2·326	2·576	3·090



INVERSE FUNCTION INTEGRALS

FUNCTION	INTEGRAL	ALTERNATIVE EXPRESSION
$\int \frac{1}{\sqrt{1-x^2}}dx$	$\sin^{-1} x$	
$\int \frac{1}{\sqrt{a^2-x^2}}dx$	$\sin^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{\sqrt{a^2-u^2}}du$	$\sin^{-1}\left(\frac{u}{a}\right)$	
$\int \frac{1}{x^2+a^2}dx$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{a^2+x^2}dx$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{u^2+a^2}du$	$\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)$	
$\int \frac{1}{\sqrt{a^2+x^2}}dx$	$\sinh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{a^2+x^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{x^2+a^2}}dx$	$\sinh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{a^2+x^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{u^2+a^2}}du$	$\sinh^{-1}\left(\frac{u}{a}\right)$	$\ln\left\lvert\frac{\left(u+\sqrt{a^2+u^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{x^2-a^2}}dx$	$\cosh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{x^2-a^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{u^2-a^2}}du$	$\cosh^{-1}\left(\frac{u}{a}\right)$	$\ln\left\lvert\frac{\left(u+\sqrt{u^2-a^2}\right)}{a}\right\rvert$
$\int \frac{1}{a^2-x^2}dx$	$\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{2a}\ln\left\lvert\frac{(a+x)}{(a-x)}\right\rvert$
$\int \frac{1}{a^2-u^2}du$	$\frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right)$	$\frac{1}{2a}\ln\left\lvert\frac{a+u}{a-u}\right\rvert$