Cork Institute of Technology

Higher Certificate in Engineering in Electrical Engineering – Award

(NFQ Level 6)

Autumn 2006

Mathematics

(Time: 3 Hours)

Instructions:

Answer ***five*** questions.

Examiners: Mr. J. Mulhare
Mr. M. Hennessy
Prof. E. McQuade

Q1 (a) Differentiate each of the following with regard to the included variable:

(i) $x^2 \sin 5x$; (ii) $\sinh^2 3t + 4\cosh 2t$; (iii) $\tanh^{-1}(4x-1)$. (12 marks)

(b) The speed v at which a telecom signal is transmitted in certain conditions is given by

$v = Kx^2 \ln(x)$ where K is a constant.

Find $\frac{dv}{dx}$ and hence show that v is max when $x = \frac{1}{\sqrt{e}}$

(16 marks)

(c) Using Implicit differentiation find the slope ($\frac{dy}{dx}$) of the circle

$$x^2 + y^2 - 2x + 6y + 7 = 0$$

(6 marks)

(d) Use parametric differentiation to find $\frac{dy}{dx}$ given that $x = \cos 3t$ and $y = \ln(t^2 - 5)$

(6 marks)

Q2 (a) Show that when $y = \cos^{-1} x$ then $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$ (6 marks)

(b) Find the values of x which satisfy the equation;

$$3\sinh x + 2\cosh x = 8$$

(10 marks)

(c) If $Z = x^4 + xe^{-y} + y^2$, determine the partial derivatives $\frac{\delta Z}{\delta x}$ and $\frac{\delta^2 z}{\delta y^2}$

(12 marks)

(d) Use the Newton-Raphson method to determine the root of the equation $e^{0.05x} = 5 - x$ for x correct to two decimal places. Using $R_1 = 6$ as your first estimate.

(12 marks)

Q3 (a) Evaluate **three** of the following integrals;

(i) $\int \cos^3 x \sin x \, dx$; ……. (substitution)

(ii) $\int \frac{3}{\sqrt{x^2 - 10x + 74}} dx$;

(iii) $\int x \cos 5x \, dx$; …… (parts)

(iv) $\int \frac{-x^2 + 51x - 40}{(x-5)(x+5)(2x-1)} dx$. …….(partial fractions) (3 @ 8 marks)

(b) Use integration to find

(i) the area enclosed by the function given below and the t-axis;

(ii) the mean value of i;

(iii) the root mean square value of i;

for the function $i = \cos t$ between $t = 0$ and $t = \frac{\pi}{2}$ …(i.e. the 1st quarter cycle)

Note: $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

(16 marks)

Q4 (a) Solve the 1st order variable separable differential equation

$$(2x+1)\frac{dy}{dx} = y - 5$$

to find the general solution.

Hence find the particular solution given that $y = 8$ when $x = 0$. (20 marks)

(b) Find the general solution for the second order differential equation;

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

to find the general solution for q in terms of t, where $L = 2,\ R = 20,\quad C = 0.02$.

Hence find its particular solution given that both $q = 0$ and $\frac{dq}{dt} = 20$ when $t = 0$.

(20 marks)

Q5 (a) The rate at which the temperature of a body cools is proportional to the difference between the temperature of the body and that of the surrounding air. The air in a closed chamber is maintained at a temperature of 10^{O}C While the temperature of a body in the chamber decreases from 240^{O}C to 160^{O}C in ten minutes.
Using this information, set up and solve a differential equation to find the temperature (T) after t minutes.
Hence, find how much longer it will take the temperature of the body to decrease to 40^{O}C.
(20 marks)

(b) Solve the 1st order linear differential equation,

$$\frac{dy}{dx} - \frac{y}{x} = \left(e^{2x} + 4\right)$$

to find the general solution. (20 marks)

Q6 (a) A switching system contains three switches X, Y and Z of different manufacture. The probability that they will fail within a year are 0.2, 0.3 and 0.4 respectively.
Calculate the probability that within a year of installation

(i) Switches X and Y will fail;
(ii) None of these switches will fail;
(iii) Only switch Z will fail;
(iv) Switch X or switch Z will fail;
(v) At least one of the switch will fail. (5 x 4 marks)

(b) The breaking pressures (bar) of a particular type of safety valves are approximately Normally distributed, with a mean of 30 bar and a standard deviation of 0.2 bar.
Find the probability that a valve chosen at random will have a breaking pressure of;

(i) Less than 30.3 bar.;
(ii) Between 30 and 30.2 bar;
(iii) Less than 29.6 bar;
(iv) More than 30.15 bar;
(v) Between 29.7 and 30.2 bar.

Between what two values does the middle 90% of breaking pressures lie? (20 marks)

7 (a) Explain what is meant by;

(i) The Identity matrix [I];

(ii) An inverse matrix. (6 marks)

If [**A**] and [**B**] are matrices such that $[\mathbf{A}]=\begin{bmatrix} 4 & 1 & 3 \\ -1 & 2 & 0 \\ 2 & -4 & 1 \end{bmatrix}$, $[\mathbf{B}]=\begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \end{bmatrix}$;

simplify the following where possible;

(i) [**A**] + [**B**]; (ii) [**A**] x [**B**]^{T}; and (iii) [**A**] - [**A**]^{T}. (12 marks)

If the matrix $[C]=\begin{pmatrix} 4 & -5 \\ 3 & 1 \end{pmatrix}$, find the inverse matrix [C]$^{-1}$. (6 marks)

(b) Using Cramers (determinants) rule at least once, solve the following network equations for i_1, i_2 *and* i_3.

$$2i_1 - i_2 + 5i_3 = 12$$
$$i_1 + 8i_2 = 10$$
$$i_2 + 6i_3 = 3$$

(16marks)

Q8 In a survey of 60 customers a mobile phone company ascertained the distances these customers were from the nearest signal mast at a particular point in time. The results were as follows.

5.4	3.2	8.7	16.4	23.8	2.5	25.4	16.3	4.2	8.4	18.6	8.4	23.2	15.4	8.5
11.7	4.2	7.8	24.3	1.3	6.4	15.8	8.2	7.8	8.7	12.2	17.4	8.5	4.3	2.4
4.3	7.7	12.3	9.5	16.4	4.7	8.9	16.3	23.5	11.2	17.9	2.4	7.3	6.5	14.3
6.4	7.2	22.5	11.8	13.2	3.6	16.7	18.4	13.2	24.1	26.4	12.7	10.3	14.2	18.4

You are required to;

(i) Arrange the above data in a grouped frequency distribution of 8 groups …..(0 to 5) , (5 to 10) etc..

(ii) Determine the mean $(\bar{x})$ and standard deviation (s) of the sample.

(iii) Construct a cumulative frequency table of the data and sketch an ogive from this table.

(iv) Using (iii) estimate the median lifetime of the bulbs and the first and third quartiles.

(v) Draw a Box-Plot of the data.

(5 @ 8 marks)

FORMULAE ETC2

FUNCTION	INTEGRAL	ALTERNATIVE EXPRESSION
$\int \frac{1}{\sqrt{1-x^2}}dx$	$\sin^{-1} x$	
$\int \frac{1}{\sqrt{a^2-x^2}}dx$	$\sin^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{\sqrt{a^2-u^2}}du$	$\sin^{-1}\left(\frac{u}{a}\right)$	
$\int \frac{1}{x^2+a^2}dx$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{a^2+x^2}dx$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{u^2+a^2}du$	$\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)$	
$\int \frac{1}{\sqrt{a^2+x^2}}dx$	$\sinh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{a^2+x^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{x^2+a^2}}dx$	$\sinh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{a^2+x^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{u^2+a^2}}du$	$\sinh^{-1}\left(\frac{u}{a}\right)$	$\ln\left\lvert\frac{\left(u+\sqrt{a^2+u^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{x^2-a^2}}dx$	$\cosh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{x^2-a^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{u^2-a^2}}du$	$\cosh^{-1}\left(\frac{u}{a}\right)$	$\ln\left\lvert\frac{\left(u+\sqrt{u^2-a^2}\right)}{a}\right\rvert$
$\int \frac{1}{a^2-x^2}dx$	$\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{2a}\ln\left\lvert\frac{(a+x)}{(a-x)}\right\rvert$
$\int \frac{1}{a^2-u^2}du$	$\frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right)$	$\frac{1}{2a}\ln\left\lvert\frac{a+u}{a-u}\right\rvert$

Differential Equations

First Order Linear;

$$ye^{\int p(x)dx} = \int e^{\int p(x)dx} q(x)dx + c\,;$$

First Order Homogeneous;

$$v = \frac{y}{x} \quad \Rightarrow \quad y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Second Order Linear Homogeneous;

Two Distinct Real Roots; $\Rightarrow \quad y = Ae^{D_1 x} + Be^{D_2 x}$;

One Real Root; $\Rightarrow \quad y = e^{Dx}\left(Ax + B\right)$;

Two complex Roots; $\Rightarrow \quad e^{px}\{A\cos qx + B\sin qx\}$

STATISTICS

Central Tendency and Dispersion

Mean of an array; $\bar{x} = \frac{\sum x}{n}$

Mean of a frequency distribution $\bar{x} = \frac{\sum fx}{\sum f}$. $\bar{x} = a + \frac{\sum fd}{\sum f}$

Standard Deviation of an array: $\sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$ or $\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

Standard Deviation of a frequency distribution $\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$

Probability distributions

Binomial Distribution; $P(r,n) = {}^{n}C_r p^r q^{n-r}$ ...mean = np ; variance = npq

Poisson Distribution; $P(x) = \frac{\lambda^x . e^{-\lambda}}{x!}$: $\lambda = mean = np$

Standard Normal Units; $x \sim N\{\mu, \sigma\} \quad \Rightarrow \quad Z = \frac{x-\mu}{\sigma}$

ETC 2

NEWTON-RAPHSON FORMULA

$$R_2 = R_1 - \frac{F(R_1)}{F'(R_1)}$$

ETC 2

NEWTON-RAPHSON FORMULA

$$R_2 = R_1 - \frac{f(R_1)}{f'(R_1)}$$

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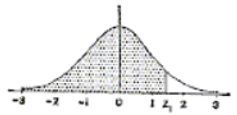
Area under the Normal Curve

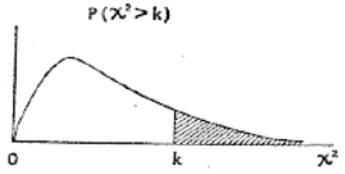
$$P(z \leqslant z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2}\, dz\,.$$



z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0·1	0·5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
0·2	0·5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
0·3	0·6179	6217	6255	6293	6331	6368	6405	6443	6480	6517
0·4	0·6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0·5	0·6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0·6	0·7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0·7	0·7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
0·8	0·7881	7910	7939	7967	7995	8023	8051	8078	8106	8133
0·9	0·8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1·0	0·8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1·1	0·8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1·2	0·8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1·3	0·9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1·4	0·9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1·5	0·9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1·6	0·9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1·7	0·9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1·8	0·9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1·9	0·9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2·0	0·9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2·1	0·9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2·2	0·9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2·3	0·9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2·4	0·9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2·5	0·99379	99396	99413	99430	99446	99461	99477	99492	99506	99520
2·6	0·99534	99547	99560	99573	99585	99598	99609	99621	99632	99643
2·7	0·99653	99664	99674	99683	99693	99702	99711	99720	99728	99736
2·8	0·99744	99752	99760	99767	99774	99781	99788	99795	99801	99807
2·9	0·99813	99819	99825	99831	99836	99841	99846	99851	99856	99861
3·0	0·99865	99869	99874	99878	99882	99886	99889	99893	99897	99900
3·1	0·99903	99906	99910	99913	99916	99918	99921	99924	99926	99929
3·2	0·99931	99934	99936	99938	99940	99942	99944	99946	99948	99950
3·3	0·99952	99953	99955	99957	99958	99960	99961	99962	99964	99965
3·4	0·99966	99968	99969	99970	99971	99972	99973	99974	99975	99976
3·5	0·99977	99978	99978	99979	99980	99981	99981	99982	99983	99983
3·6	0·99984	99985	99985	99986	99986	99987	99987	99988	99988	99989
3·7	0·99989	99990	99990	99990	99991	99991	99992	99992	99992	99992
3·8	0·99993	99993	99993	99994	99994	99994	99994	99995	99995	99995
3·9	0·99995	99995	99996	99996	99996	99996	99996	99996	99997	99997

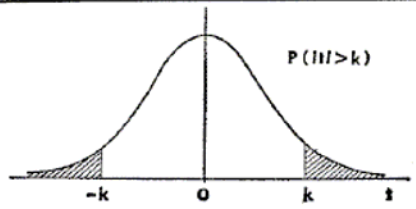
χ^2-D ÁILEADH — χ^2-DISTRIBUTION

	·99	·95	·50	·20	·10	·05	·025	·01
1	·0002	·0039	·45	1·64	2·71	3·84	5·02	6·63
2	·020	·103	1·39	3·22	4·61	5·99	7·38	9·21
3	·115	·352	2·37	4·64	6·25	7·81	9·35	11·34
4	·30	·71	3·36	5·99	7·78	9·49	11·14	13·28
5	·55	1·15	4·35	7·29	9·24	11·07	12·83	15·09
6	·87	1·64	5·35	8·56	10·64	12·59	14·45	16·81
7	1·24	2·17	6·35	9·80	12·02	14·07	16·01	18·48
8	1·65	2·73	7·34	11·03	13·36	15·51	17·53	20·09
9	2·09	3·33	8·34	12·24	14·68	16·92	19·02	21·67
10	2·56	3·94	9·34	13·44	15·99	18·31	20·48	23·21
11	3·05	4·57	10·34	14·63	17·28	19·68	21·92	24·72
12	3·57	5·23	11·34	15·81	18·55	21·03	23·34	26·22
13	4·11	5·89	12·34	16·98	19·81	22·36	24·74	27·69
14	4·66	6·57	13·34	18·15	21·06	23·68	26·12	29·14
15	5·23	7·26	14·34	19·31	22·31	25·00	27·49	30·58
16	5·81	7·96	15·34	20·47	23·54	26·30	28·85	32·00
17	6·41	8·67	16·34	21·61	24·77	27·59	30·19	33·41
18	7·02	9·39	17·34	22·76	25·99	28·87	31·53	34·81
19	7·63	10·12	18·34	23·90	27·20	30·14	32·85	36·19
20	8·26	10·85	19·34	25·04	28·41	31·41	34·17	37·57
21	8·90	11·59	20·34	26·17	29·62	32·67	35·48	38·93
22	9·54	12·34	21·34	27·30	30·81	33·92	36·78	40·29
23	10·20	13·09	22·34	28·43	32·01	35·17	38·08	41·64
24	10·86	13·85	23·34	29·55	33·20	36·42	39·36	42·98
25	11·52	14·61	24·34	30·68	34·38	37·65	40·65	44·31
26	12·20	15·38	25·34	31·79	35·56	38·89	41·92	45·64
27	12·88	16·15	26·34	32·91	36·74	40·11	43·19	46·96
28	13·57	16·93	27·34	34·03	37·92	41·34	44·46	48·28
29	14·26	17·71	28·34	35·14	39·09	42·56	45·72	49·59
30	14·95	18·49	29·34	36·25	40·26	43·77	46·98	50·89
40	22·16	26·51	39·34	47·27	51·81	55·76	59·34	63·69
50	29·71	34·76	49·33	58·16	63·17	67·50	71·42	76·15
60	37·48	43·19	59·33	68·97	74·40	79·08	83·30	88·38
70	45·44	51·74	69·33	79·71	85·53	90·53	95·02	100·43
80	53·54	60·39	79·33	90·41	96·58	101·88	106·63	112·33
90	61·75	69·13	89·33	101·05	107·57	113·15	118·14	124·12
100	70·06	77·93	99·33	111·67	118·50	124·34	129·56	135·81



t-DÁILEADH — t-DISTRIBUTION

	20	10	5	2	1	0·2
1	3·078	6·314	12·706	31·821	63·657	318·310
2	1·886	2·920	4·303	6·965	9·925	22·327
3	1·638	2·353	3·182	4·541	5·841	10·215
4	1·533	2·132	2·776	3·747	4·604	7·173
5	1·476	2·015	2·571	3·365	4·032	5·893
6	1·440	1·943	2·447	3·143	3·707	5·208
7	1·415	1·895	2·365	2·998	3·499	4·785
8	1·397	1·860	2·306	2·896	3·355	4·501
9	1·383	1·833	2·262	2·821	3·250	4·297
10	1·372	1·812	2·228	2·764	3·169	4·144
11	1·363	1·796	2·201	2·718	3·106	4·025
12	1·356	1·782	2·179	2·681	3·055	3·930
13	1·350	1·771	2·160	2·650	3·012	3·852
14	1·345	1·761	2·145	2·624	2·977	3·787
15	1·341	1·753	2·131	2·602	2·947	3·733
16	1·337	1·746	2·120	2·583	2·921	3·686
17	1·333	1·740	2·110	2·567	2·898	3·646
18	1·330	1·734	2·101	2·552	2·878	3·610
19	1·328	1·729	2·093	2·539	2·861	3·579
20	1·325	1·725	2·086	2·528	2·845	3·552
21	1·323	1·721	2·080	2·518	2·831	3·527
22	1·321	1·717	2·074	2·508	2·819	3·505
23	1·319	1·714	2·069	2·500	2·807	3·485
24	1·318	1·711	2·064	2·492	2·797	3·467
25	1·316	1·708	2·060	2·485	2·787	3·450
26	1·315	1·706	2·056	2·479	2·779	3·435
27	1·314	1·703	2·052	2·473	2·771	3·421
28	1·313	1·701	2·048	2·467	2·763	3·408
29	1·311	1·699	2·045	2·462	2·756	3·396
30	1·310	1·697	2·042	2·457	2·750	3·385
40	1·303	1·684	2·021	2·423	2·704	3·307
60	1·296	1·671	2·000	2·390	2·660	3·232
120	1·289	1·658	1·980	2·358	2·617	3·160
∞	1·282	1·645	1·960	2·326	2·576	3·090



Aonad faid: méadar (m)
Aonad mais: cileagram (kg)
Aonad fórsa: Niútan (N) = kgm/s^2
Aonad oibre: giúl (J) = Nm
Aonad cumhachta: vata (W) = J/s

Gluaiseacht i líne faoi luasghéarú tairiseach: $v = u + ft$; $s = ut + \frac{1}{2}ft^2$; $v^2 = u^2 + 2fs$

Fuinneamh Poitéinsiúil: mgh. Fuimneamh cinéatach (obair déanta): $\frac{1}{2}mv^2$.
I gcóras imchoimeádach: Fuinneamh poitéinsiúil + fuinneamh cinéatach = méad tairiseach.

Móiminteam chaithnín: $\vec{mu}$
Spreagadh fórsa = athrú san móiminteam.
Comhéifeacht chúitimh (comhéifeacht leaisteachais):

$$e = -\left(\frac{\text{luas gaolmhar indhiaidh iombhualadh}}{\text{luas gaolmhar roimh iombhualadh}}\right)$$

Luasghéarú láraimsitheach: $\frac{v^2}{r} = \omega^2 r$

Dlí Hooke le haghaidh téada leaistigh: $p = kx$ (p an fórsa, x an sineadh, tairiseach don téad é k).

Meánlár:

Stua, gath r; uillinn 2θ ag an lár: $\frac{r \sin\theta}{\theta}$ ón lárphointe.

Teascóg diosca; gath r, uillinn 2θ: $\frac{2}{3}r\,\frac{\sin\theta}{\theta}$ ón lárphointe.

Lann triantánach: $\frac{1}{3}$ ón mbonn feadh an mheánlíne.

Meáchanlár:
Leathsféar, gath r: $\frac{3}{8}r$ ón lárphointe.
Sceall leathsféarach, gath r: $\frac{1}{2}r$ ón lárphointe.
Drón-chón ciorclach, airde h: $\frac{1}{4}h$ ón bhonn.

Brú ag pointe i leacht: ρgh.
Sá ar dhromcla átá báite: achar × brú ag an meánlár.

Móimintí na táimhe:
Bata aonfhoirmeach, fad $2l$: lár: $\frac{1}{3}ml^2$; foirceann: $\frac{4}{3}ml^2$
Fonsa aonfhoirmeach, gath r: lár: mr^2; lárlíne: $\frac{1}{2}mr^2$
Diosca aonfhoirmeach, gath r: lár: $\frac{1}{2}mr^2$; lárline: $\frac{3}{4}mr^2$
Dlúth-sféar aonfhoirmeach, gath r: lár líne: $\frac{2}{5}mr^2$