Cork Institute of Technology

Higher Certificate in Engineering in Electrical Engineering – Award

(NFQ Level 6)

Summer 2007

Mathematics

(Time: 3 Hours)

Instructions: Answer ***five*** questions.

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Q1 (a) State the amplitude, period and frequency of the following functions and sketch their graphs

(i) $20\sin 10\pi t$;

(ii) $50\sin(20\pi t + \pi/4)$

(b) Find the turning points and points of inflection of the graph of the function ;

$$f(x) = x^3 - 6x^2 - 15x + 10$$

Sketch the graphs of the functions $f(x)$ and $f'(x)$ against x.

(10 marks)

(c) Differentiate each of the following with regard to the included variable:

(i) $2\sinh 5x + x^2 \cosh 5x$;

(ii) $\cos^{-1} 2x + \sin^{-1}(3x - 1)$ (10 marks)

(d) The coordinates of a prolate cycloid after t seconds are given by

$x = 2t - \pi \sin t$ and $y = 2 - \pi \cos t$

Use parametric differentiation to find $\frac{dy}{dx}$ in terms of t. (10 marks)

Q2 (a) Prove the hyperbolic identity that $\cosh^2 x - \sinh^2 x = 1$. (8 marks)

(b) A cable hangs in the catenary $y = c\cosh\frac{x}{c}$ where C is the minimum height and its length for a span of $2a$ is $2L$ where $L = c\sinh\frac{a}{c}$.

Find the span and the height of two equal supports for a cable of length 100 m. where the minimum height is 5m. (12 marks)

(c) Electrical power P is given by $P = \frac{E^2}{R}$ where E is the voltage and R is resistance. Using partial differentiation, approximate the maximum percentage error in calculating power, P, where the maximum possible percentage error in E and R are 2% and 3 % respectively.

(8 marks)

(d) Use the Newton-Raphson method to determine the root of the equation $50e^{-x} = 0.5x - 4$ for x correct to two decimal places. Use $r_1 = 7$ as you first estimate.

(12 marks)

Q3 (a) Evaluate four of the following integrals

(i) $\int\left(8x^5 - 4\cos 2x + 7e^{-3x} - \frac{3}{x}\right)dx$

(ii) $\int \sin\ x.\cos^5 x dx$ ……..by substitution

(iii) $\int_3^5 \frac{5x+2}{(x-2)(2x-1)}dx$ ……using partial fractions.9

(iv) $\int x\sin 2x dx$ ……by parts

(v) $\int \frac{4}{\sqrt{16x^2 - 25}}dx$ (4 x 8 marks)

(b) Determine the mean value of the function $i = 5\sin^2 10\pi t$ for values of t between 0 and 0.1

<u>Note</u>: $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ (8 marks)

Q4 (a) By means of separating the variables find a general solution for the differential equation

$$(2y)\frac{dy}{dx} = \left(y^2 + 4\right)(x + 5)$$

Find also a particular solution given that $y = 10$ when $x = 0$

(20 marks)

(b) A tank contains one thousand litres of a water solution in which 200kg of salt is dissolved initially. Distilled water is poured into the tank at a rate of 100 litres per minute and the well stirred solution is withdrawn at the same rate. Let y kg. equal the amount of salt in the tank after t minutes. At what rate is salt being removed from the tank after t minutes?

Set up a differential equation which displays this information.

Solve this differential equation to find an expression for y in terms of t.

Find how long it takes to reduce the amount of salt in the tank to 100 kg.

(20 marks)

Q5 (a) Find the general solution for the second order differential equation;

$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 20y = 0$$

Hence find its particular solution given that $y = 0$ and $\frac{dy}{dx} = 13$ when $x = 0$.

(20 marks)

(b) Write the following equation as a first order linear differential equation;

$$(x + 1)\frac{dy}{dx} + y = e^{2x}$$

Solve the equation to find the general solution.

Determine the particular solution given that y=10 .when x=0.

(20 marks)

Q6 (a). Solve the following network simultaneous equations for i_1 by using Cramer's rule (Determinants).

$$3i_1 + 4i_2 - i_3 = 14$$
$$2i_1 - 5i_2 - 5i_3 = 19$$
$$i_1 - 4i_2 + 2i_3 = -10$$

(16 marks)

(b) Given the following matrices A and B, determine, where possible,

(i) **C=A-B,** (ii) **D = AB,** and (iii) **E = BA**

$$\mathbf{A} = \begin{pmatrix} 4 & 1 \\ -1 & 3 \\ -2 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 5 & -1 \\ 2 & 4 \end{pmatrix}$$

Find $\mathbf{B^{-1}}$, i.e. the inverse of the matrix **B.**

(10 marks)

(c) Show that $Q = \frac{1}{26}\begin{pmatrix} 9 & -11 & -3 \\ 3 & 5 & -1 \\ -7 & -3 & 11 \end{pmatrix}$ is the inverse of the matrix $P = \begin{pmatrix} 2 & 5 & 1 \\ -1 & 3 & 0 \\ 1 & 4 & 3 \end{pmatrix}$

Hence find the values of A,B and C which satisfy the equations

$$2A + 5B + C = 12$$
$$-1A + 3B \quad = 16$$
$$A + 4B + 3C = 23$$

(14 marks)

Q7 The distances traveled (km) to calls by a maintenance engineer on a series of sixty working days were as follows;

146	123	184	163	172	186	143	106	108	117
113	138	156	208	153	175	165	166	173	191
146	142	104	135	148	154	158	163	113	162
174	153	157	196	144	172	192	142	162	185
198	176	156	164	159	163	162	128	178	153
184	165	147	194	188	146	108	195	203	155

You are required to;

(i) Arrange the data in a grouped frequency distribution of six groups i.e. (100-119) etc;

(ii) Determine the mean and standard deviation of the sample;

(iii) Construct an ogive of the data;

(iv) Estimate the median and the first and third quartiles of the sample;

(v) Sketch a box-plot of the data.

(5 x 8 marks)

Q8 (a) The probabilities that ***machine A, machine B and machine C*** will be performing a useful function in four years time are 0.85, 0.6 and 0.7 respectively.

Given that ***Machines A,B and C*** are independent, draw a Venn diagram depicting these probabilities.

Calculate the probability that in four years time;

(i) ***Machines A*** and ***B*** will be operating a useful function.

(ii) none will be operating a useful function.

(iii) only ***machine A*** will be operating a useful function.

(iv) ***Machine A or Machine C*** will be operating.

(v) at least one of the machines will be operating a useful function.

(5 x 4 marks)

(b) Oil is stored in containers with a nominal volume of 50 litres. The volumes are in fact normally distributed with a mean of 50.15 litres and a standard deviation of 0.1 litres.

You are required to determine the probability that a container chosen at random contains;

(i) Less than 50.24 ltr
(ii) Less than 50 ltr.
(iii) Between 50.25 and 50.4 Ltr
(iv) Between 50 and 50.4 ltr.
(v) More than 49.98

Below what value do the lowest 10% of volumes lie?

Between what two values do the middle 90% of volumes lie? (5 x 4 marks)

FORMULAE ETC2

FUNCTION	INTEGRAL	ALTERNATIVE EXPRESSION
$\int \frac{1}{\sqrt{1-x^2}}dx$	$\sin^{-1}x$	
$\int \frac{1}{\sqrt{a^2-x^2}}dx$	$\sin^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{\sqrt{a^2-u^2}}du$	$\sin^{-1}\left(\frac{u}{a}\right)$	
$\int \frac{1}{x^2+a^2}dx$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{a^2+x^2}dx$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\int \frac{1}{u^2+a^2}du$	$\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)$	
$\int \frac{1}{\sqrt{a^2+x^2}}dx$	$\sinh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{a^2+x^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{x^2+a^2}}dx$	$\sinh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{a^2+x^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{u^2+a^2}}du$	$\sinh^{-1}\left(\frac{u}{a}\right)$	$\ln\left\lvert\frac{\left(u+\sqrt{a^2+u^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{x^2-a^2}}dx$	$\cosh^{-1}\left(\frac{x}{a}\right)$	$\ln\left\lvert\frac{\left(x+\sqrt{x^2-a^2}\right)}{a}\right\rvert$
$\int \frac{1}{\sqrt{u^2-a^2}}du$	$\cosh^{-1}\left(\frac{u}{a}\right)$	$\ln\left\lvert\frac{\left(u+\sqrt{u^2-a^2}\right)}{a}\right\rvert$
$\int \frac{1}{a^2-x^2}dx$	$\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{2a}\ln\left\lvert\frac{(a+x)}{(a-x)}\right\rvert$
$\int \frac{1}{a^2-u^2}du$	$\frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right)$	$\frac{1}{2a}\ln\left\lvert\frac{a+u}{a-u}\right\rvert$

Differential Equations

First Order Linear;

$$ye^{\int p(x)dx} = \int e^{\int p(x)dx} q(x)dx + c\,;$$

First Order Homogeneous;

$$v = \frac{y}{x} \quad \Rightarrow \quad y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Second Order Linear Homogeneous;

Two Distinct Real Roots; $\Rightarrow \quad y = Ae^{D_1 x} + Be^{D_2 x}$;

One Real Root; $\Rightarrow \quad y = e^{Dx}(Ax + B)$;

Two complex Roots; $\Rightarrow \quad e^{px}(A\cos qx + B\sin qx)$

STATISTICS

Central Tendency and Dispersion

Mean of an array; $\bar{x} = \frac{\sum x}{n}$

Mean of a frequency distribution $\bar{x} = \frac{\sum fx}{\sum f}$. $\bar{x} = a + \frac{\sum fd}{\sum f}$

Standard Deviation of an array: $\sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$ or $\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$

Standard Deviation of a frequency distribution $\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$

Probability distributions

Binomial Distribution; $P(r,n) = {}^nC_r p^r q^{n-r}$ .. mean $= np$; variance $= npq$

Poisson Distribution; $P(x) = \frac{\lambda^x . e^{-\lambda}}{x!}$: $\lambda =$ mean $= np$

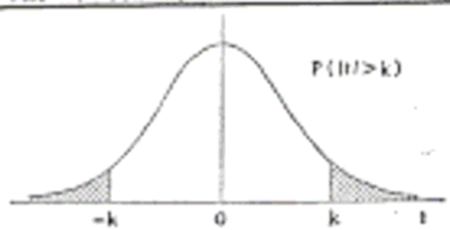
Standard Normal Units; $x - N(\mu, \sigma) \quad \Rightarrow \quad Z = \frac{x-\mu}{\sigma}$

ETC 2

NEWTON-RAPHSON FORMULA

$$R_2 = R_1 - \frac{f(R_1)}{f'(R_1)}$$

t-DÁILEADH					t-DISTRIBUTION
ν \ P%	20	10	5	2	1	0·2
1	3·078	6·314	12·706	31·821	63·657	318·310
2	1·886	2·920	4·303	6·965	9·925	22·327
3	1·638	2·353	3·182	4·541	5·841	10·215
4	1·533	2·132	2·776	3·747	4·604	7·173
5	1·476	2·015	2·571	3·365	4·032	5·893
6	1·440	1·943	2·447	3·143	3·707	5·208
7	1·415	1·895	2·365	2·998	3·499	4·785
8	1·397	1·860	2·306	2·896	3·355	4·501
9	1·383	1·833	2·262	2·821	3·250	4·297
10	1·372	1·812	2·228	2·764	3·169	4·144
11	1·363	1·796	2·201	2·718	3·106	4·025
12	1·356	1·782	2·179	2·681	3·055	3·930
13	1·350	1·771	2·160	2·650	3·012	3·852
14	1·345	1·761	2·145	2·624	2·977	3·787
15	1·341	1·753	2·131	2·602	2·947	3·733
16	1·337	1·746	2·120	2·583	2·921	3·686
17	1·333	1·740	2·110	2·567	2·898	3·646
18	1·330	1·734	2·101	2·552	2·878	3·610
19	1·328	1·729	2·093	2·539	2·861	3·579
20	1·325	1·725	2·086	2·528	2·845	3·552
21	1·323	1·721	2·080	2·518	2·831	3·527
22	1·321	1·717	2·074	2·508	2·819	3·505
23	1·319	1·714	2·069	2·500	2·807	3·485
24	1·318	1·711	2·064	2·492	2·797	3·467
25	1·316	1·708	2·060	2·485	2·787	3·450
26	1·315	1·706	2·056	2·479	2·779	3·435
27	1·314	1·703	2·052	2·473	2·771	3·421
28	1·313	1·701	2·048	2·467	2·763	3·408
29	1·311	1·699	2·045	2·462	2·756	3·396
30	1·310	1·697	2·042	2·457	2·750	3·385
40	1·303	1·684	2·021	2·423	2·704	3·307
60	1·296	1·671	2·000	2·390	2·660	3·232
120	1·289	1·658	1·980	2·358	2·617	3·160
∞	1·282	1·645	1·960	2·326	2·576	3·090



MATAMAITIC FHEIDHMEACH

Aonad faid: méadar (m)
Aonad mais: cileagram (kg)
Aonad fórsa: Niútan (N) = kgm/s²
Aonad oibre: giúl (J) = Nm
Aonad cumhachta: vata (W) = J/s

Gluaiseacht i líne faoi luasghéarú tairiseach: $v = u + ft$; $s = ut + \frac{1}{2}ft^2$; $v^2 = u^2 + 2fs$

Fuinneamh Poitéinsiúil: mgh. Fuinneamh cinéiteach (obair déanta): $\frac{1}{2}mv^2$.
I bpróiseas inchoimeádach: Fuinneamh poitéinsiúil + fuinneamh cinéiteach = méid tairiseach.

Móiminteam cháithnín: $m\vec{v}$
Spreagadh fórsa = athrú san móiminteam.
Comhéifeacht chúitimh (comhéifeacht leaisteachais):

$$e = -\left(\frac{\text{luas gaolmhar indiaidh comhbhualadh}}{\text{luas gaolmhar roimh comhbhualadh}}\right)$$

Luasghéarú lárthreoch: $\frac{v^2}{r} = \omega^2 r$

Dlí Hooke le haghaidh téada leaisteacha: $p = kx$ (p an fórsa, x an síneadh, tairiseach don téad é k).

Meánlár:
Stua, ga r; uillinn 2θ ag an lár: $\frac{r \sin\theta}{\theta}$ ón lárphointe.

Teascóg diosca; ga r, uillinn 2θ: $\frac{2}{3}r\frac{\sin\theta}{\theta}$ ón lárphointe.

Lann triantánach: $\frac{1}{3}$ ón mbonn feadh an airmheánlíne.

Meáchanlár:
Leathsféar, ga r: $\frac{3}{8}r$ ón lárphointe.
Sceall leathsféarach, ga r: $\frac{1}{2}r$ ón lárphointe.
Dronn-chón ciorclach, airde h: $\frac{1}{4}h$ ón mbonn.

Brú ag pointe i leacht: ρgh.
Sá ar dhromchla atá báite: achar × brú ag an meánlár.

Móiminti na táimhe:
Bata aonfhoirmeach, fad $2l$: lár: $\frac{1}{3}ml^2$; foirceann: $\frac{4}{3}ml^2$
Fonsa aonfhoirmeach, ga r: lár: mr^2; lárlíne: $\frac{1}{2}mr^2$
Diosca aonfhoirmeach, ga r: lár: $\frac{1}{2}mr^2$; lárlíne: $\frac{1}{4}mr^2$
Dlúth-sféar aonfhoirmeach, ga r: lár líne: $\frac{2}{5}mr^2$

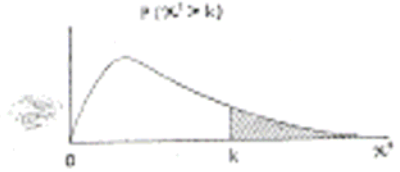
Area under Normal Curve

Area under the Normal Curve

$$P(z \leq z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2}\, dz .$$

z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0·1	0·5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
0·2	0·5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
0·3	0·6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
0·4	0·6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0·5	0·6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0·6	0·7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0·7	0·7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
0·8	0·7881	7910	7938	7967	7995	8023	8051	8078	8106	8133
0·9	0·8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1·0	0·8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1·1	0·8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1·2	0·8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1·3	0·9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1·4	0·9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1·5	0·9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1·6	0·9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1·7	0·9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1·8	0·9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1·9	0·9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2·0	0·9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2·1	0·9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2·2	0·9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2·3	0·9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2·4	0·9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2·5	0·99379	99396	99413	99430	99446	99461	99477	99492	99506	99520
2·6	0·99534	99547	99560	99573	99585	99598	99609	99621	99632	99643
2·7	0·99653	99664	99674	99683	99693	99702	99711	99720	99728	99736
2·8	0·99744	99752	99760	99767	99774	99781	99788	99795	99801	99807
2·9	0·99813	99819	99825	99831	99836	99841	99846	99851	99856	99861
3·0	0·99865	99869	99874	99878	99882	99886	99889	99893	99897	99900
3·1	0·99903	99906	99910	99913	99916	99918	99921	99924	99926	99929
3·2	0·99931	99934	99936	99938	99940	99942	99944	99946	99948	99950
3·3	0·99952	99953	99955	99957	99958	99960	99961	99962	99964	99965
3·4	0·99966	99968	99969	99970	99971	99972	99973	99974	99975	99976
3·5	0·99977	99978	99978	99979	99980	99981	99981	99982	99983	99983
3·6	0·99984	99985	99985	99986	99986	99987	99987	99988	99988	99989
3·7	0·99989	99990	99990	99990	99991	99991	99992	99992	99992	99992
3·8	0·99993	99993	99993	99994	99994	99994	99994	99995	99995	99995
3·9	0·99995	99995	99996	99996	99996	99996	99996	99996	99997	99997

ν	·99	·95	·50	·20	·10	·05	·025	·01
1	·0002	·0039	·45	1·64	2·71	3·84	5·02	6·63
2	·020	·103	1·39	3·22	4·61	5·99	7·38	9·21
3	·115	·352	2·37	4·64	6·25	7·81	9·35	11·34
4	·30	·71	3·36	5·99	7·78	9·49	11·14	13·28
5	·55	1·15	4·35	7·29	9·24	11·07	12·83	15·09
6	·87	1·64	5·35	8·56	10·64	12·59	14·45	16·81
7	1·24	2·17	6·35	9·80	12·02	14·07	16·01	18·48
8	1·65	2·73	7·34	11·03	13·36	15·51	17·53	20·09
9	2·09	3·33	8·34	12·24	14·68	16·92	19·02	21·67
10	2·56	3·94	9·34	13·44	15·99	18·31	20·48	23·21
11	3·05	4·57	10·34	14·63	17·28	19·68	21·92	24·72
12	3·57	5·23	11·34	15·81	18·55	21·03	23·34	26·22
13	4·11	5·89	12·34	16·98	19·81	22·36	24·74	27·69
14	4·66	6·57	13·34	18·15	21·06	23·68	26·12	29·14
15	5·23	7·26	14·34	19·31	22·31	25·00	27·49	30·58
16	5·81	7·96	15·34	20·47	23·54	26·30	28·85	32·00
17	6·41	8·67	16·34	21·61	24·77	27·59	30·19	33·41
18	7·02	9·39	17·34	22·76	25·99	28·87	31·53	34·81
19	7·63	10·12	18·34	23·90	27·20	30·14	32·85	36·19
20	8·26	10·85	19·34	25·04	28·41	31·41	34·17	37·57
21	8·90	11·59	20·34	26·17	29·62	32·67	35·48	38·93
22	9·54	12·34	21·34	27·30	30·81	33·92	36·78	40·29
23	10·20	13·09	22·34	28·43	32·01	35·17	38·08	41·64
24	10·86	13·85	23·34	29·55	33·20	36·42	39·36	42·98
25	11·52	14·61	24·34	30·68	34·38	37·65	40·65	44·31
26	12·20	15·38	25·34	31·79	35·56	38·89	41·92	45·64
27	12·88	16·15	26·34	32·91	36·74	40·11	43·19	46·96
28	13·57	16·93	27·34	34·03	37·92	41·34	44·46	48·28
29	14·26	17·71	28·34	35·14	39·09	42·56	45·72	49·59
30	14·95	18·49	29·34	36·25	40·26	43·77	46·98	50·89
40	22·16	26·51	39·34	47·27	51·81	55·76	59·34	63·69
50	29·71	34·76	49·33	58·16	63·17	67·50	71·42	76·15
60	37·48	43·19	59·33	68·97	74·40	79·08	83·30	88·38
70	45·44	51·74	69·33	79·71	85·53	90·53	95·02	100·43
80	53·54	60·39	79·33	90·41	96·58	101·88	106·63	112·33
90	61·75	69·13	89·33	101·05	107·57	113·15	118·14	124·12
100	70·06	77·93	99·33	111·67	118·50	124·34	129·56	135·81



37