CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Winter Examinations 2015/16

Module Title: Technological Maths 2

Module Code: **MATH6015**

School: School of Mechanical, Electrical and Process Engineering

Programme Titles:
B Eng (Hons) in Building Energy Systems – Year 2
B Eng (Hons) in Sustainable Energy Engineering – Year 2
Bachelor of Engineering in Biomedical Engineering – Year 2
Bachelor of Engineering in Building Services Engineering – Year2
Bachelor of Engineering in Mechanical Engineering – Year 2

Programme Codes:
EBENS_8_Y2
ESENT_8_Y2
EBIME_7_Y2
EBSEN_7_Y2
EMECH_7_Y2

External Examiner: Dr James Cruickshank

Internal Examiners: **Dr C. Carroll, Dr M. Lishchynska, Mr J.P. McCarthy**

Instructions: **Answer ALL questions.**

Duration: 2 HOURS

Sitting: Winter 2015

Exam Requirements: Mathematics Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.

If in doubt please contact an Invigilator.

1. (a) Differentiate $f(x) = 3x^2 - 2x + 5$ from first principles.

   No marks will be awarded if any other method is used.

   [7 Marks]

   (b) Find the slope of the curve $y = x\,e^x$ at the point where $x = 0.2$. Give your answer correct to two decimal places.

   [4 Marks]

   (c) Differentiate the following functions by rule:

   i. $y = 2x^5 + \sqrt{x} + \ln(2x) - 8$.

   [5 Marks]

   ii. $y = e^{-3x}(x^2 + 7)^5$.

   [5 Marks]

   iii. $z = \dfrac{\sin(\pi t)}{t^2}$.

   [4 Marks]

2. (a) The angular displacement in radians, $\theta(t)$, of a particle in circular motion is given by

$$\theta(t) = 3\sin\left(2t - \frac{\pi}{3}\right),$$

for t between 0 and π seconds.

i. Find the angular velocity, $\omega(t)$.

[3 Marks]

ii. Find the angular acceleration, $\alpha(t)$.

[3 Marks]

iii. When is the **angular acceleration** first equal to zero?

[5 Marks]

(b) Suppose that the temperature, T, of a device is changing in time, t, according to:

$$T(t) = -0.6t^2 + 9t + 9.$$

The temperature is measured in $^\circ$C and t in minutes. Find the maximum temperature of the device.

[4 Marks]

(c) An engineer has been asked to produce a part for a machine. The part needs to be in the shape of a hollow cylinder, closed at one end and the total surface area of the part equal 1.5 m^2.

i. Find an expression for the height, h, of the cylinder in terms of the radius, r.

[3 Marks]

ii. Find the dimensions of the cylinder that maximise the volume. Give your answers for h and r correct to four decimal places.

[7 Marks]

3. (a) Find the following:

i. $\displaystyle\int \left(2x^6 + \frac{4}{x} - 2e^{-0.5x} + 10\right)\, dx.$

[5 Marks]

ii. $\displaystyle\int_0^{\pi/10} 4\cos(5t)\, dt.$

[3 Marks]

iii. $\displaystyle\int (x+3)(x^2+6x)^4\, dx.$

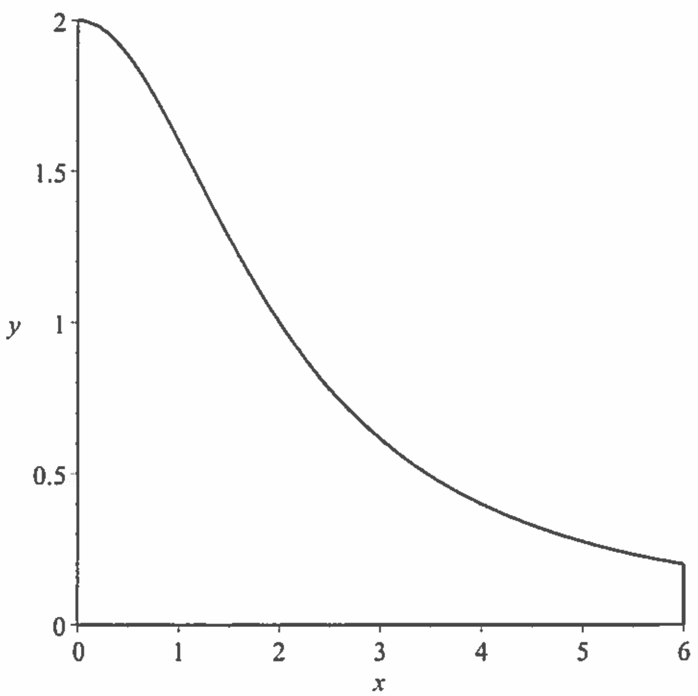
[6 Marks]

iv. $\displaystyle\int \frac{x-5}{(x-2)(x-3)}\, dx.$

[5 Marks]

(b) Find the area enclosed between the curve $y = \dfrac{8}{x^2+4}$, the coordinate axes and the line $x = 6$.

Give your answer correct to the nearest whole number.



[6 Marks]

4. (a) A force of $F(x) = 100 - 0.12x^2$, measured in Newtons, raises up a leaking bucket (where x is the distance from the ground). Find the work done in raising the leaking bucket from the ground to a height of $x = 20$ m.

[7 Marks]

(b) Using a formula, or otherwise, find the mean value of the function

$$f(t) = \cos t$$

in the region $t = 0$ to $t = 2\pi$.

[4 marks]

Show that the root mean square value of $f(t) = \cos t$ over this region is $\frac{1}{\sqrt{2}}$.

[6 marks]

(c) The bending moment, $M(x)$, at a point x on a beam satisfies:

$$\frac{dM}{dx} = 54 - 18x.$$

Solve this differential equation for $M(x)$ if $M = 0$ when $x = 0$.

[8 marks]

Useful Formulae

$$V = \pi \int y^2 \, dx$$

$$W = \int F \, dx$$

$$\overline{y} = \frac{1}{b-a} \int y \, dx$$

$$y_{\text{RMS}} = \sqrt{\frac{1}{b-a} \int y^2 \, dx}$$