Cork Institute of Technology

Special Mathematics Examination for Engineering Degree Entry

June 2003

Time: 2 hours and 30 minutes

PAPER 1 (300 marks)

Attempt **SIX** questions.
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

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1. (a) Find the coefficient of the x^3 term in the product

$$\left(1 + x - 2x^2\right)\left(x - 3x^2 + 4x^3\right).$$

(b) Show that $x = 2$ is a root of the equation

$$x^3 - 4x^2 + 3x + 2 = 0.$$

Find the other two roots.

(c) The equation $4x^2 + kx + 9 = 0$, $k < 0$, $k \in \mathbf{R}$, has two equal roots.

(i) Find the value of k.

(ii) Find the roots of the equation.

(iii) Find the equation whose roots are twice the roots of the given equation.

2. (a) Solve $x = \sqrt{3x + 4}$.

(b) **(i)** Find the range of values of $x \in \mathbf{R}$ for which $\dfrac{x-3}{x+2} \leq 1$.

(ii) Let $f(x) = \dfrac{1}{x^n}$ for all $x \in \mathbf{R}$ and $n \in \mathbf{N}$.

Simplify the product $\left(f(x)\right)\left(f\left(\dfrac{1}{x}\right)\right)$.

(c) The roots of the equation $x^2 - 2bx + c = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = 4b^2 - 2c$.

(ii) Write $\alpha^4 + \beta^4$ in terms of b and c.

3. (a) Find the value of x and the value of y if

$$\begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}.$$

(b) Let $z = 2 - 3i$ and $w = 4 + 5i$, where $i^2 = -1$.

(i) Write down $\bar{z}$, the conjugate of z.

(ii) Verify that $\overline{z + w} = \bar{z} + \bar{w}$.

(iii) Find the value of k and the value of t such that

$$z + kw = ti, \qquad k, t \in \mathbf{R}.$$

(c) **(i)** Prove that if

$$z = r(\cos\theta + i\sin\theta), \text{ then } z^n = r^n(\cos n\theta + i\sin n\theta).$$

(ii) Write $(\cos 5\pi + i\sin 5\pi)^{\frac{1}{3}}$ in the form $\dfrac{p + iq}{r}$, $\quad p, q, r \in \mathbf{R}$.

4. (a) Find the sum of the first 20 terms of the arithmetic series $5 + 2 - 1 \ldots$

(b) $u_r = \dfrac{1}{\sqrt{r+1} + \sqrt{r}}$ for all $r \in \mathbf{N}$.

(i) Show that $u_r = \sqrt{r+1} - \sqrt{r}$.

(ii) Evaluate, in terms of n, $\sum_{r=1}^{n} u_r$.

(iii) Evaluate $\sum_{r=1}^{99} u_r$.

(c) $u_{n+1} = \dfrac{u_n}{2}$, $\quad u_1 = 1$ and $n \in \mathbf{N}$.

(i) Find u_2, u_3, and u_4.

(ii) Find S_n, the sum of the first n terms of the series $u_1 + u_2 + u_3 + \ldots$.

(iii) Find the least value of n for which $S_n > 1.9$.

5. (a) $\log x^2 = \log 9 - 4\log 2$.

Find x, $x > 0$.

(b) **(i)** Find the middle term in the expansion of $\left(5 - \frac{1}{5}\right)^8$.

Simplify your answer.

(ii) Solve $10^{2x} - 11.10^x + 10 = 0$.

(c) Prove by induction that $\frac{1}{n!} < \frac{1}{2^{n-1}}$, $n > 2$, $n \in \mathbf{N}$.

6. (a) Differentiate $x + \frac{1}{x}$ with respect to x.

(b) **(i)** If $y = (x^3 - 2)^4$, find $\frac{dy}{dx}$ at $x = 1$.

(ii) Differentiate $x\sqrt{1-x^2}$ with respect to x and simplify your answer.

(c) Let $x = \sin t - t\cos t$ and $y = \cos t + t\sin t$.

Show that $\frac{dy}{dx} = \frac{\cos t}{\sin t}$.

For what value of t is $\frac{dy}{dx} = 1$, $0 < t < \frac{\pi}{2}$.

7. (a) Find the equation of the tangent to the curve

$$y^2 = 3xy - 2x$$

at the point (1, 2).

(b) **(i)** If $y = \sqrt{\dfrac{\sin x}{1 - \cos x}}$, evaluate $\dfrac{dy}{dx}$ at $x = \dfrac{\pi}{2}$.

(ii) Show that $x^3 + x - 1 = 0$ has a root between 0 and 1.

Take $x_1 = 1$ as the first approximation and use the Newton-Raphson method to show that $0.7 < x_2 < 0.8$ where x_2 is the second approximation.

(c) Let $f(x) = \dfrac{x-1}{x+2}$, where $x \neq -2$ and $x \in \mathbf{R}$.

(i) Show that $f(x)$ is increasing for $x \in \mathbf{R}$ and $x \neq -2$.

(ii) Find the co-ordinates of the points at which $f(x)$ cuts the axes.

(iii) Find the equations of the asymptotes of the curve.

(iv) Draw a sketch of the curve.

8. (a) Find **(i)** $\int (x^2 - 1)dx$ **(ii)** $\int \cos x \, dx$..

(b) Evaluate **(i)** $\int_1^2 \dfrac{1}{x+1} dx$ **(ii)** $\int_0^{\frac{\pi}{4}} \cos^3 2x \sin 2x \, dx$.

(c) Sketch the graphs of

$$y = 2x \quad \text{and} \quad y = x^3$$

for $-2 \leq x \leq 2$, $x \in \mathbf{R}$.

Find the area of the region in the first quadrant enclosed by the two graphs.

Write down the total area enclosed between the two graphs and give a reason for your answer.