Cork Institute of Technology

Special Mathematics Examination for Engineering Degree Entry

June 2003

Time: 2 hours and 30 minutes

PAPER 2 (300 marks)

Attempt **FIVE** questions from Section **A** and **ONE** question from Section **B.**
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

SECTION A

Answer FIVE questions from this section

1. (a) The parametric equations

$$x = 2 + 4\cos\theta, \qquad y = -3 + 4\sin\theta$$

define a circle.

Find the Cartesian equation of the circle.

(b) The line T has equation $3x + 4y + 13 = 0$.

The circle C has equation $x^2 + y^2 + 4y + 3 = 0$.

(i) Show that T is a tangent to C.

(ii) The circle K has equation $(x-6)^2 + (y-6)^2 = 121$.

T is also a tangent to K.

Show that the circles C and K touch internally.

(iii) Show that T is perpendicular to the line joining the centres of the two circles.

(c) Find the equations of the two circles of radius 2 which touch both axes and have their centres on the line $x + y = 0$.

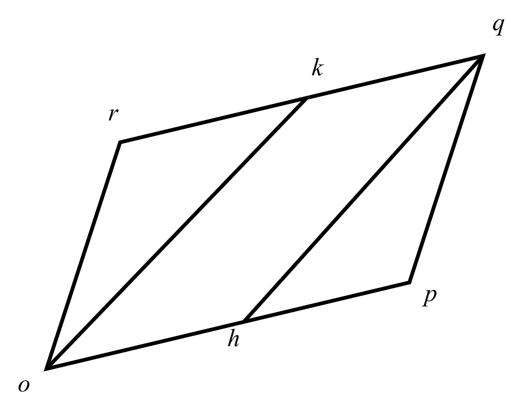
2. (a) $\vec{a} = -\vec{i} + 2\vec{j}, \quad \vec{b} = 7\vec{i} - 4\vec{j}.$

(i) Express $\overrightarrow{ab}$ in the form $x\vec{i} + y\vec{j},\ x, y \in \mathbf{R}$.

(ii) Find $|\overrightarrow{ab}|$.

(b) *opqr* is a parallelogram, where *o* is the origin.

h and *k* are the midpoints of the two sides $[op]$ and $[rq]$.



(i) Express $\overrightarrow{hq}$ and $\overrightarrow{ok}$ in terms of $\vec{p}$ and $\vec{r}$.

(ii) Deduce that *ohqk* is a parallelogram.

Let $\vec{r} = \vec{i} + 2\vec{j}$ and $\vec{p} = 3\vec{i} + \vec{j}$.

(iii) Show that $|\angle rop| = 45°$.

(iv) Calculate the area of *opqr*.

(v) $\vec{v} = \alpha\vec{r} + \beta\vec{p}$. Find the scalars α and β if $\vec{v} = 3\vec{i} + 3\vec{j}$.

3. (a) f is the transformation $(x, y) \to (x', y')$ where $x' = 4x - 2y$ and $y' = x + 3y$.

$o(0, 0)$ and $a(6, 4)$ are two points.

Find m, the midpoint of $[oa]$.

f maps $a \to a'$ and $m \to m'$.

Show that m' is the midpoint of $[oa']$.

(b) Prove that the angle θ between the two lines with slopes m_1 and m_2 is given by

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2} .$$

Show that the two lines which each make an angle of $45°$ with the line $6x + y = 3$ are perpendicular to each other.

(c) $a(5, 1)$, $b(2, 6)$ and $c(x, y)$ are the vertices of a triangle.

Find the equations of the two lines for which the area of the triangle $abc = 11$.

The point $c(x, y)$ lies on the line $x = 3$ and is in the fourth quadrant.

Find the value of x and the value of y.

4. (a) If $\sin A = \frac{1}{4}$, $0° \leq A \leq 90°$, find the value of $\tan A$.

(b) In the triangle abc,

$$|ab| = 5, \quad |ac| = 3 \quad \text{and} \quad |\angle abc| = 27°.$$

(i) Find two possible values for $|\angle acb|$, correct to the nearest degree.

(ii) Given that $|\angle acb| < 90°$, find the area of the triangle abc, correct to the nearest integer.

(c) **(i)** Write $\cos 2A$ in terms of $\cos A$.

Hence, or otherwise, solve the equation

$$\cos A = \cos 2A, \quad 0 \leq A \leq 2\pi.$$

(ii) Prove that

$$\sin^3 X - \cos^3 X = (\sin X - \cos X)(1 + \sin X \cos X).$$

5. (a) Find the values of θ for which

$$2\sin\theta = -1, \qquad 0 \le \theta \le 2\pi.$$

(b) Prove that

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}.$$

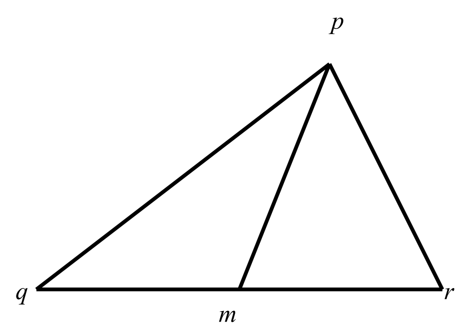
If $\cos 2A = \frac{12}{13}$, find the two values of $\tan A$.

(c) In the triangle pqr,

m is the midpoint of $[qr]$.

Let $|qr| = 2a, \quad |pr| = b, \quad |pq| = c,$

$|pm| = k \quad$ and $\quad |\angle pmr| = \theta$.



(i) Using the cosine rule, express b^2 in terms of a, k and θ.

(ii) Show that $b^2 + c^2 = 2a^2 + 2k^2$.

(iii) Find k, if $|qr| = 8$, $|pr| = 5$ and $|pq| = 6$.

Give your answer correct to one decimal place.

6. (a) A code is in the form of a consonant, followed by a digit in the range 1 to 9, followed by a vowel.

(i) How many different codes are possible?

(ii) How many of these codes contain the digit 5?

(b) Solve the difference equation

$$u_{n+2} = 3u_{n+1} - 4u_n \quad \text{where} \quad n \ge 0$$

given that $u_0 = 5$ and $u_1 = 5$.

Evaluate u_7.

(c) **(i)** In how many ways can the letters of the word COUNTER be arranged?

(ii) How many of the arrangements begin with a consonant and end with a vowel?

(iii) In how many of the arrangements are all the consonants together?

(iv) In how many of the arrangements are no two consonants together?

7. (a) The probability that Paul swims a 100 m race in a given time interval is 0.6.

The probability that Orla swims the same race in the same given time interval is 0.8.

Both Paul and Orla swim the race.

Find the probability that at least one of them finishes within the given time interval.

(b) A game consists of throwing a red die and a blue die together.

The faces of the red die are numbered 1, 2, 3, 4, 5, 6.

The faces of the blue die are numbered 1, 1, 3, 3, 5, 5.

Find the probability that in one throw

(i) the sum of the numbers thrown is less than 5

(ii) the same number is thrown on each die

(iii) the sum of the numbers thrown is even

(iv) the number on the red die is greater than the number on the blue die.

(c) The numbers x_1, x_2, x_3 have mean $\overline{x}$ and standard deviation σ.

(i) Find the mean and standard deviation of the numbers $x_1 - 1,\ x_2 - 1,\ x_3 - 1$ in terms of $\overline{x}$ and σ.

(ii) Find the mean and standard deviation of the set $\{0, 5, 10, 15, 20\}$.

Deduce the mean and standard deviation of the set $\{550, 555, 560, 565, 570\}$.

SECTION B

Answer ONE question from this section.

8. (a) Use the ratio test to show that $\sum_{n=1}^{\infty} \frac{2n}{n!}$ is convergent.

(b) Evaluate $\int_0^{\frac{\pi}{2}} x^2 \cos 2x dx$

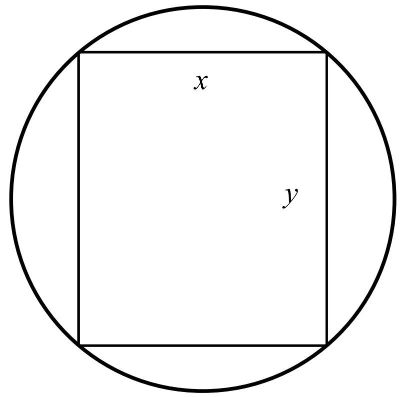
(c) A rectangle of length x and width y is inscribed in a circle.

The radius of the circle is $2\sqrt{2}$.

(i) Express y in terms of x.

(ii) Find the value of x for which the rectangle has maximum area.

(iii) Calculate this maximum area.



9. (a) In a school 60% of the students are girls and 40% are boys.

10% of the girls are less than 170 cm tall and 5% of the boys are less than 170 cm tall.

A student is picked at random.

What is the probability the student is less than 170 cm tall?

(b) A multiple choice test consists of 10 questions. Each question has four possible answers. A student answers the questions using guesswork only. What is the probability

(i) of getting no correct answers

(ii) of getting exactly four correct answers?

Give your answers correct to two places of decimals.

(c) A machine in a bottling plant automatically fills bottles. The manufacturers claim that the bottles will have a mean content of 500 ml and a standard deviation of 18 ml.

The Company selects 36 bottles at random and measures their contents. Their mean contents is 505 ml.

At the 5% level of significance is this result consistent with the manufacturer's claim?

10. (a) The set {1, 3, 5, 7} is a group under multiplication modulo 8.

(i) Write down the Cayley Table for the group.

(ii) State the identity element and write down the inverse of each element.

(b) The set {*a, b, c, d*}, * is a group.

Part of the Cayley Table for the group is shown

*	*a*	*b*	*c*	*d*
a	*b*	*d*	*a*	
b	*d*	*c*	*b*	
c	*a*			
d				

(i) Complete the table.

(ii) Write down the identity element of the group.

(iii) What is the inverse of *a* and of *b*?

(iv) Is {*c*, *d*}, * a subgroup?

Explain your answer.

(v) Is the group a commutative group? Explain your answer.

(vi) Verify that $(a * b) * d = a * (b * d)$.

(vii) Is the group isomorphic to the group {1, 3, 5, 7}, × mod 8, in part **(i)** above.

Explain your answer.

11. (a) Find the equation of the ellipse with centre (0, 0), eccentricity $\frac{1}{3}$ and one focus at (3, 0).

(b) *L* and *M* are perpendicular lines and *f* is a similarity transformation.

Prove $f(L) \perp f(M)$.

(c) The tangents from an external point *p* to a circle *S* touch the circle at *q* and *r*.

Prove that *qr* is the polar of *p*.