Cork Institute of Technology

Special Mathematics Examination for Engineering Degree Entry

June 2006

Time: 2 hours and 30 minutes

PAPER 1 (300 marks)

Attempt **SIX** questions.
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

1. (a) Solve the simultaneous equations

$$2x - \frac{y}{3} = 0$$

$$\frac{3x}{2} + \frac{5y}{3} = \frac{23}{4}.$$

(b) **(i)** Solve for x: $|x-5| < 7$, where $x \in \mathbf{R}$.

(ii) Given that $a^3 + b^3 = 28$ and $a + b = 4$, find the value of ab.

(c) The equations $x^2 + px + 1 = 0$ and $x^2 + x + q = 0$ have a common root.

(i) Show that this root is $\frac{q-1}{p-1}$.

(ii) Deduce that $(q-1)^2 = (p-1)(1-pq)$.

2. (a) Simplify fully $\frac{x^2 + \sqrt{x^5}}{\sqrt{x}}$.

(b) $(x-1)$ and $(x+2)$ are factors of $g(x) = 2x^3 + hx^2 + kx - 6$.

(i) Find the value of h and the value of k.

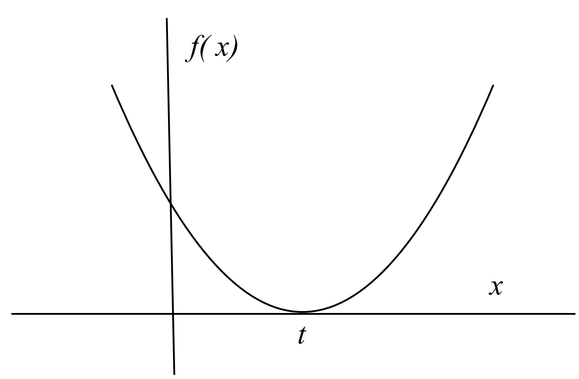
(ii) Find the roots of $g(x) = 0$.

(c) The graph of $f(x) = ax^2 + bx + c$, $a > 0$, is shown.

The curve touches the x-axis at the point t.

(i) Find the co-ordinates of t in terms of a and b.

(ii) Given that $c = 4a$, find the value of a, of b and of c.



3. (a) Let $k = \frac{1}{i}\left(i^6 + i^7 + i^8\right)$, where $i = \sqrt{-1}$ and $k \in \mathbf{Z}$.
Find k.

(b) Let $A = \begin{pmatrix} 7 & 5 \\ 4 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

(i) Find $A + B$.

(ii) Verify that $AB \neq BA$.

(iii) Find the matrix X such that $AX = B$.

(c) **(i)** Prove by induction that

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta, \quad n \in \mathbf{N_0}.$$

(ii) Hence, or otherwise, evaluate $(1+i)^{16}$.

4. (a) Evaluate $\sum_{n=1}^{50}(2n-5)$.

(b) **(i)** Given that $u_n = 2 - \frac{1}{2^{n+1}}$, show that $u_n - u_{n-1} > 0$.

(ii) The first three terms of a geometric series are

$$(2x-1) + \frac{1}{2} + \frac{x}{4} + \ldots \qquad \text{where } x > 0.$$

Find the sum to infinity of the series.

(c) Simplify $\left(1 + \frac{p}{q}\right)^4 + \left(1 - \frac{p}{q}\right)^4$.

Deduce the value of $\left(\sqrt{2}+\sqrt{3}\right)^4 + \left(\sqrt{2}-\sqrt{3}\right)^4$.

5. (a) Find the 100th term of the arithmetic sequence $-3, \quad 1, \quad 5, \quad \Lambda$.

(b) **(i)** Solve for x: $9^x + 3 = 4 \times 3^x$.

(ii) Solve for x:

$$\log_{10}(3x+4) = 2\log_{10} x$$

and verify your solution.

(c) Prove by induction that 4 is a factor of $5^{2n+3} - 1$ for $n \in \mathbf{N}$.

6. (a) Let $y = \dfrac{x^2 - 1}{x}$.

Find the value of $\dfrac{dy}{dx}$ when $x = 1$.

(b) **(i)** Differentiate $x \sin x^2$ with respect to x.

(ii) Given that $y = \log_e\left(1 + 2x^2\right)$ find $\dfrac{dy}{dx}$ at $x = \dfrac{1}{2}$.

(c) Let $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$.

Given that

$$2\sqrt{x}\left(\frac{dy}{dx}\right) + y = ke^{\sqrt{x}}$$

find k.

7. (a) Find, from first principles, the derivative of x^2 with respect to x.

(b) The equation of a curve is $y = x^2 e^x$.

(i) Show that $x = 0$ is a root of the equation.

(ii) Find the value of x at which the curve has a local maximum turning point and the value of x at which the curve has a local minimum turning point.

(c) The equation of a curve is $y = \dfrac{x}{x+4}$, where $x \neq -4$.

(i) Find the equations of the asymptotes to the curve.

(ii) Show that the curve is increasing for $x \in \mathbf{R}$, $x \neq -4$.

(iii) Draw a sketch of the curve.

(iv) Find the equation of the tangent to the curve at the point (0, 0).

8. (a) Find **(i)** $\int (5x^3 - 3)\, dx$ **(ii)** $\int \frac{1}{x}\, dx$.

(b) **(i)** Evaluate $\int_0^1 (x+1)e^{x^2+2x}\, dx$.

(ii) Find the constant of integration given that $\int (3x^2 - 4x + 2)dx = 1$ when $x = -2$.

(c) **(i)** Evaluate $\int_{\pi/2}^{\pi} \sin 3\theta \sin 2\theta \, d\theta$.

(ii) Use integration methods to derive the formula for the volume of a sphere of radius r.