Cork Institute of Technology

Special Mathematics Examination for Engineering Degree Entry

June 2007

Time: 2 hours and 30 minutes

PAPER 1 (300 marks)

Attempt **SIX** questions.
Each question carries 50 marks.

WARNING: Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

1. (a) Simplify fully $\dfrac{x}{x^2-1}-\dfrac{1}{x+1}$.

(b) **(i)** Given that $x^2-2x-1\le 0$, show that $1-\sqrt{2}\le x\le 1+\sqrt{2}$.

(ii) Write as a natural number $2^{\frac{1}{3}}\left(2^{\frac{2}{3}}+4^{\frac{1}{3}}\right)$.

(c) **(i)** Let $f(x)=ax^3+bx^2+cx+d$.
Prove that if $(x-k)$ is a factor of the polynomial $f(x)$, then $f(k)=0$.

(ii) If k is a root of the equation

$$2x^3-5x^2-5x+2=0$$

prove that $\dfrac{1}{k}$ is also a root.

2. (a) Solve the simultaneous equations

$$x=2y-1$$
$$xy+y^2=2.$$

(b) **(i)** Find the range of values of $x\in\mathbf{R}$ for which

$$\frac{3x+2}{x+4}<2,\quad x\neq -4.$$

(ii) Show that for $x\in\mathbf{N}$, x^2+3x+2 is always even.

(c) Let $f(x)=ax^2+(a+d)x+d=0$.

(i) Show that the roots of the equation $f(x)=0$ are real.

(ii) Given that the roots of the equation $f(x)=0$ are equal, find the roots.

3. **(a)** Let $M = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$.

(i) Find $2M$.

(ii) Find M^2.

(b) Write $\frac{4-2i}{3+i}$ in the form $x+iy$, where $x,\ y \in \mathbf{R}$ and $i^2 = -1$.

Hence, or otherwise, find the value of $k \in \mathbf{R}$ such that $\left(\frac{4-2i}{3+i}\right)^4 = k$.

(c) Let $z = -1 + i\sqrt{3}$.

(i) Find the square roots of z and write your answers in the form $x+iy$.

(ii) Evaluate $\frac{64}{z^6}$.

4. **(a)** Write the recurring decimal $0.131313\ldots$ as an infinite geometric series and hence as a fraction.

(b) The sequence $u_1,\ u_2,\ u_3 \cdots$ has $u_n = 2(3^n) + 4^n$.

(i) Find u_1.

(ii) Show that $u_{n+2} = 7u_{n+1} - 12u_n$.

(c) Show that $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$.

Deduce $\sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)}$.

5. (a) Find, in terms of n, the sum of the first n terms of the arithmetic series $2 + 4 + 6 \ldots$.

(b) **(i)** Solve $\sqrt{x+7} = 7 - \sqrt{x}$.

(ii) The first two terms in the expansion of

$$(1+kx)^4 + (1-kx)^4$$

are $2 + 3x^2$.

Find the values of k.

(c) Prove by induction that $5 + 5^2 + 5^3 + \cdots \; 5^n = \frac{5}{4}(5^n - 1)$.

6. (a) Differentiate $\tan^{-1} 2x$ with respect to x.

(b) **(i)** Differentiate $\log(1+\cos x)$ with respect to x.

(ii) Let $y = \sqrt{x} + \frac{1}{x}$.

Find $\frac{dy}{dx}$ at $x = 4$.

(c) Let $y = e^{-x}\cos x$.

(i) Show that $\frac{dy}{dx} = -e^{-x}(\cos x + \sin x)$.

(ii) Find $\frac{d^2y}{dx^2} + 2\frac{dy}{dx}$.

Write your answer in terms of y.

7. **(a)** Let $s = (2t-3)^4$ where s is in metres and t is in seconds.
Find the rate of change of s with respect to t when $t = 2$.

(b) Let $f(x) = x^3 - 6x^2 + 9x$.

(i) Evaluate $f(0)$.

(ii) Find the co-ordinates of the turning points of the curve $y = f(x)$ and state which is a local maximum and which is a local minimum turning point.

(iii) Draw a sketch of the curve $y = f(x)$.

(c) Find the equation of the tangent to the curve

$$x^2y + xy^2 = 6$$

at the point (2, 1).

8. **(a)** Find **(i)** $\int (2-4x^3)\,dx$ **(ii)** $\int \cos 3x \, dx$.

(b) **(i)** Evaluate $\int_1^2 \frac{1}{3x-2}\,dx$.

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \sin^2 x \, dx$.

(c) The diagram shows the graphs of the curve $f(x) = 6x - x^2$ and the line $g(x) = 2x$.
Calculate the area of the region enclosed by the curves $y = f(x)$ and $y = g(x)$.