

Cork Institute of Technology

CIT Mathematics Examination 2014

Paper 1

Tuesday 19 August 2014, 10:00–12:30

Time: 2 hours, 30 minutes

Instructions

Answer **ALL FIVE** questions.

Each question is worth 20 marks.

Total marks available: 100 marks.

- The *Formulae and Tables* booklet (State Examinations Commission) is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.
- Answers should be given in simplest form, where relevant.

Q1

- (a) A prize fund is divided as follows:

the first prize is half of the fund,
the second prize is three-quarters of the first prize,
the third prize is what remains.

If the value of the third prize is €150, what is each of the other two prizes worth?

[3 marks]

- (b) Light travels at a speed of approximately 3×10^8 metres per second. How many kilometres will light travel in 20 minutes? Express your answer in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{N}$.

[3 marks]

- (c) Joe is booking four tickets for a show in New York. Each ticket costs \$72, and the ticket agent charges a 5% commission charge per ticket.

If the current exchange rate is €1 = \$1.36, calculate the total amount (in euro) which Joe must pay for the four tickets. Give your answer correct to the nearest cent.

[4 marks]

- (d) Aoife invested €3000 for three years at compound interest. The rate of interest was 2% per annum for the first two years, and increased to 2.5% per annum for the third year.

Calculate the total value of the investment after the third year.

[4 marks]

- (e) Let $z_1 = 2 + 5i$ and $z_2 = 3 - 4i$, where $i^2 = -1$.

- (i) Write each of the following numbers in the form $a + bi$, where $a, b \in \mathbb{R}$:

- $z_1 + z_2$
- $z_1 z_2$

- (ii) Plot the number $-\bar{z}_1$ on an Argand diagram (where \bar{z} denotes the complex conjugate of z).

[6 marks]

Q2

- (a) Temperature can be measured in degrees Celsius ($^{\circ}C$) and degrees Fahrenheit ($^{\circ}F$). The relationship between these two scales of temperature is given by

$$C = \frac{5(F - 32)}{9}$$

where C denotes temperature in degrees Celsius and F denotes temperature in degrees Fahrenheit.

- (i) Calculate C when $F = 149$.
- (ii) Express F in terms of C .
- (iii) Hence or otherwise, convert $40^{\circ}C$ to degrees Fahrenheit.

[4 marks]

- (b) Express each of the following in the form 5^n , where $n \in \mathbb{Q}$.

(i) 125 (ii) $\frac{1}{25}$ (iii) $\sqrt{5}$ (iv) 1

[3 marks]

- (c) Solve each of the following equations for x :

- (i) $10^{2x-1} = 1$
- (ii) $2^x = 0.5$
- (iii) $9^{x+1} = \frac{1}{3^{2x}}$

[6 marks]

- (d) Solve for x and y :

$$\begin{aligned}x + y &= 1 \\x^2 - y^2 &= 5\end{aligned}$$

[3 marks]

- (e) A rectangular flower bed measures $(x + 5)$ metres by $(x - 2)$ metres. Its area equals 18 m^2 .

Find the dimensions of the flower bed.

[4 marks]

Q3

- (a) Three schemes for number plates are being evaluated.
- Under the first scheme, each number plate contains three letters of the alphabet and two digits. No letter or digit can be repeated, and the first digit cannot be zero.
 - Under the second scheme, each number plate contains two letters of the alphabet and three digits. As with the first scheme, no letter or digit can be repeated, and the first digit cannot be zero.
 - Under the third scheme, each number plate contains two letters of the alphabet and three digits, with the first digit being a non-zero digit, as in the second scheme. However, while no letter can be repeated, a digit may be repeated.

How many different number plates can be manufactured under each of the schemes?

[4 marks]

- (b) Two hundred students, girls and boys, took an examination recently. The grades achieved are recorded in the following table:

Grade	A	B	C	D	E
Number of Girls	8	32	20	12	8
Number of Boys	16	40	36	20	8

If a student is selected at random, find the probability that the student is

- a girl;
- a boy who got grade B;
- a boy or girl who got grade A;
- a girl who got grade C or D.

[5 marks]

Question 3 continued overleaf

Question 3 continued

(c) The diagram shows two wheels.

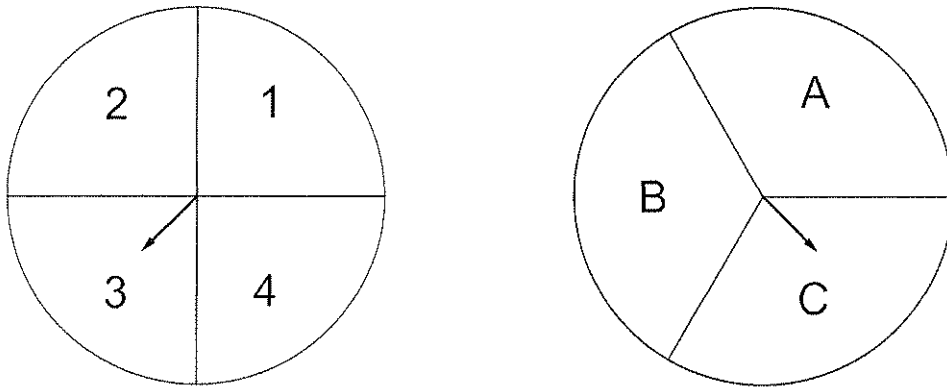
The first wheel is divided into four equal segments numbered 1, 2, 3 and 4.

The second wheel is divided into three equal segments labelled A , B and C .

A game consists of spinning the two wheels and noting the segments that stop at the arrows. For example, the outcome shown is $(3, B)$.

- (i) Write down all the possible outcomes.
- (ii) What is the probability that the outcome is $(2, C)$?
- (iii) What is the probability that the outcome is a vowel with an odd number?

[5 marks]



(d) Nine employees work in a garage: six mechanics and three salespersons. The mean wage of the mechanics is €600 per week while the mean wage of the salespersons is €570 per week.

- (i) Find the total wage bill for a week.
- (ii) Find the mean wage of the nine employees.
- (iii) In September 2014, the garage will employ a service manager, who will be paid €650 per week. If all other wages remain unchanged, calculate the mean weekly wage in September 2014.

[6 marks]

Q4

(a) The line l has equation $2x - y + 4 = 0$.

- (i) The line l intersects the x -axis at the point P . Find the co-ordinates of the point P .
- (ii) Another line, k , passes through the origin and is perpendicular to the line l . Find an equation for the line k .
- (iii) Determine the point at which the lines k and l intersect.

[6 marks]

(b) A circle c has centre $(0, 0)$ and radius 5 units.

- (i) Show the circle c on a co-ordinate diagram.
- (ii) Find the equation of c .
- (iii) Prove that the point $(2, 3)$ is inside c .

[3 marks]

(c) The length of the sides of a right-angled triangle are, in increasing order, 8 cm, 15 cm, and x cm.

- (i) Find x , the length of the hypotenuse of this triangle.
- (ii) Find the value of the smallest angle of this triangle. Give your answer correct to the nearest degree.

[4 marks]

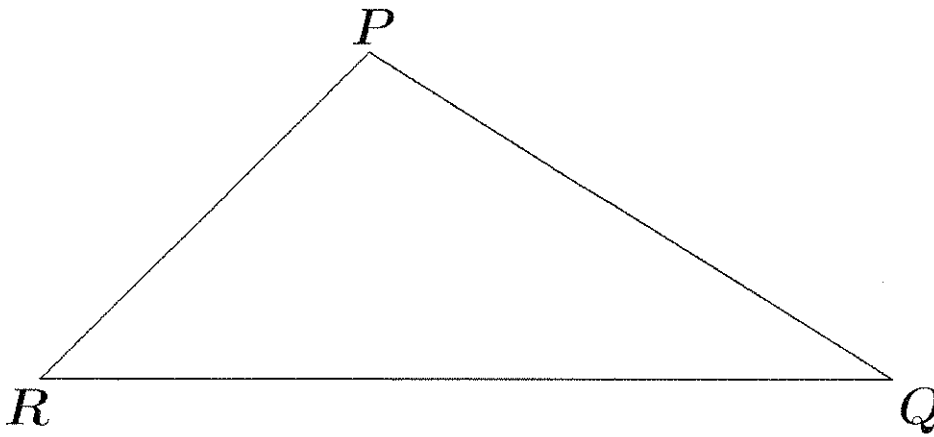
Question 4 continued overleaf

Question 4 continued

(d) In the diagram below, $|PQ| = 8$ cm, $|QR| = 11$ cm, and $|PR| = 6$ cm.

- (i) Calculate $|\angle PQR|$, correct to the nearest degree.
- (ii) Hence calculate the obtuse angle $|\angle QPR|$, correct to the nearest degree.

[7 marks]



Q5

(a) Let $f(x) = 5 - x^2 + 4x$ where $x \in \mathbb{R}$.

- (i) Find the value of $f(0)$;
- (ii) Solve the equation $f(x) = 0$;
- (iii) Find $f'(x)$, the derivative of $f(x)$.
- (iv) Hence find the x -coordinate of the local maximum point of the curve $y = f(x)$.
- (v) Use your answers to parts (i) to (iv) of this question to sketch the graph of f in the domain $-2 \leq x \leq 6$.

[7 marks]

(b) Let $g(x) = x^3 - 2x^2 + 3$, for $x \in \mathbb{R}$.

- (i) Find $g(-2)$.
- (ii) Find $g'(-2)$, i.e. the value of $\frac{dg}{dx}$ at the point where $x = -2$.
- (iii) Hence find an equation for the tangent to the curve of $g(x)$ at the point where $x = -2$.

[5 marks]

(c) A jet is moving along an airport runway. At the instant it passes a marker, it begins to accelerate for take-off. From the time the jet passes the marker, its distance from the marker is given by

$$s = 2t^2 + 5t$$

where s is in metres and t is in seconds.

- (i) Find the speed of the jet at the instant it passes the marker ($t = 0$).
- (ii) Find the speed of the jet three seconds after it passes the marker.
- (iii) In order to take off, the speed of the jet must be at least 85 metres per second.
 - How long does it take the jet to reach this speed?
 - How far is the jet from the marker at that time?

[8 marks]