Cork Institute of Technology

CIT Mathematics Exam 2014

Paper 2

Tuesday 19 August 2014, 14:00–16:30

Time: 2 hours, 30 minutes

Instructions

Answer **ALL FIVE** questions.

Each question is worth 20 marks.
Total marks available: 100 marks.

- The Formulae and Tables booklet (State Examinations Commission) is available.
- Marks will be lost if all necessary work is not clearly shown.
- Answers should include the appropriate units of measurement, where relevant.
- Answers should be given in simplest form, where relevant.

Q1

(a) A complex number written in rectangular form is given by

$$z = 1 - i$$

(i) Express z in polar form.

(ii) Using De Moivre's Theorem, show that $z^7 = 8 + 8i$.

[7 marks]

(b) Three numbers are in arithmetic sequence. Their sum is 30 and their product is 510. Find the three numbers.

[4 marks]

(c) (i) Find the sum to infinity of the geometric sequence

$$1, \frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{10000} \cdots$$

(ii) Hence express the recurring decimal $0.1\dot{3}$ in the form $\frac{p}{q}$ where $p, q \in \mathbb{N}$.

[4 marks]

(d) A solution is cooling down uniformly according to the law

$$\theta(t) = 300e^{-0.02t}$$

where $\theta(t)$ is the temperature in $°C$ after t minutes.

(i) What is the initial temperature of the solution?

(ii) What is the temperature of the solution after 5 minutes?

(iii) How long will it take for the temperature of the solution to reach $100°C$?

[5 marks]

Q2

(a) Show that $x = 3$ is a root of the cubic equation $x^3 - x^2 - x - 15 = 0$.
Find all other roots of this equation.

[5 marks]

(b) Solve the following simultaneous equations for x, y and z:

$$\begin{aligned} 3x - y + 3z &= 1 \\ x + 2y - 2z &= -1 \\ 4x - y + 5z &= 4 \end{aligned}$$

[5 marks]

(c) Solve each of the following equations for x:

(i) $\log_2(x+1) = 3, \quad x > -1$

(ii) $\log_5(x+1) + \log_5(x-1) = \log_5 8, \quad x > 1$

(iii) $\log_2 x + \dfrac{12}{\log_2 x} = 7, \quad x > 0$

[7 marks]

(d) Find the set of all real values of x for which $|1 - 4x| < 5$.

[3 marks]

Q3

(a) A letter is selected at random from the following set of letters:

$$P, R, O, B, A, B, I, L, I, T, Y$$

(a) Find the probability that the letter selected is a consonant;

(b) Find the probability that the letter selected is B, if it is already known that the letter selected is a consonant.

[4 marks]

(b) A book club has 10 members, made up of 4 men and 6 women. A committee of 3 members of the club is to be formed to organise the club's Christmas dinner.

(i) How many different committees can be formed?

(ii) Find the probability that the committee has exactly two women.

(iii) Find the probability that all members of the committee are women.

(iv) Find the probability that there are at least two women on the committee.

[6 marks]

(c) A fair die is rolled five times.

(i) Find the probability that a six is obtained on the first two rolls only.

(ii) Find the probability that exactly two sixes are obtained.

(iii) Find the probability that exactly two even numbers are obtained.

[5 marks]

(d) The manufacturer of Seacláid chocolate bars claims that the weight of the bars is normally distributed with a mean of 125 grams (g) and a standard deviation of 5 g.

(i) What percentage of Seacláid chocolate bars weigh more than than 135 g?

(ii) A shop buys a box of 200 Seacláid chocolate bars. How many bars in the box are expected to weigh less than 118 g?

[5 marks]

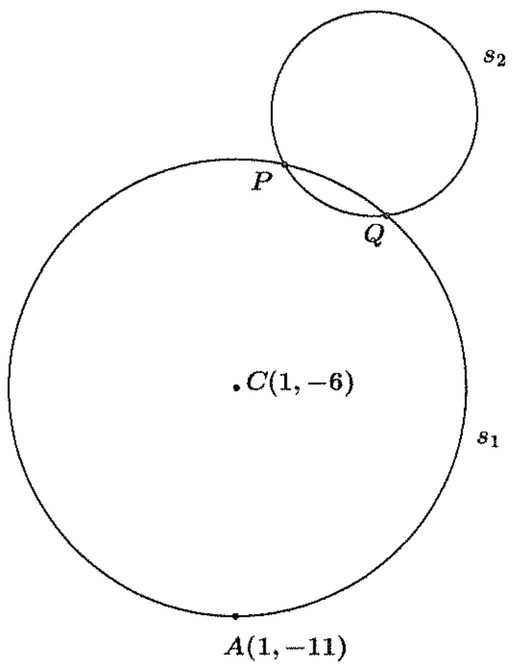
Q4

(a) $A(1, -11)$ and $C(1, -6)$ are two points. The line segment $[A, C]$ is a radius for the circle s_1, the larger of the two circles shown in the diagram below.

(i) Find the equation of the circle s_1.

(ii) The smaller circle, $s_2 : x^2 + y^2 - 8x + 11 = 0$, intersects the circle s_1 at the points P and Q.
Find an equation for the line PQ.

[4 marks]



(b) A is an acute angle such that $\tan A = \frac{8}{15}$.
Without evaluating A, find

(i) $\cos A$

(ii) $\sin 2A$

[4 marks]

Question 4 continued overleaf

Question 4 continued

(c) Solve the equation $\tan 2\theta = -\frac{1}{\sqrt{3}}$, where $0 \leq \theta < 360°$ is in degrees.

[5 marks]

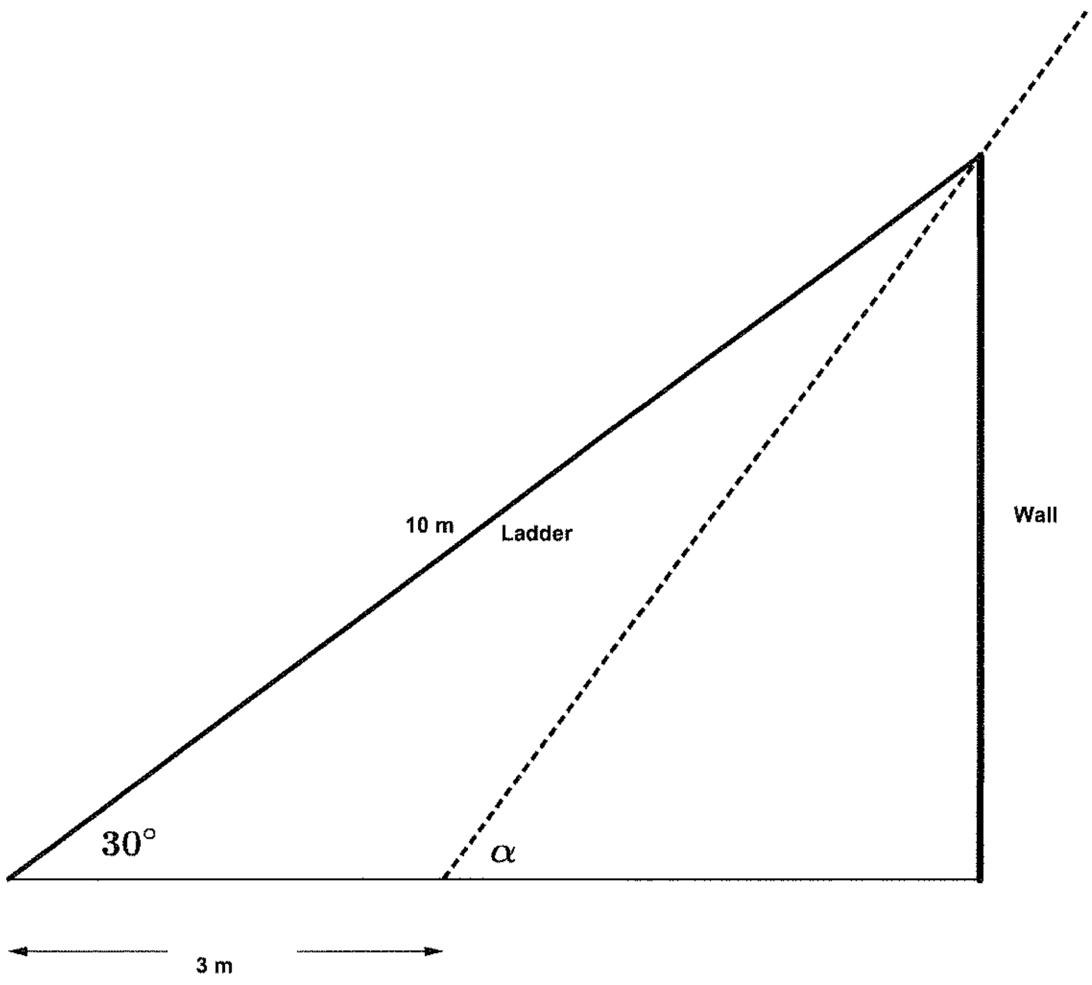
(d) A ladder of length 10 metres (m) is placed against a vertical wall so that its end just lies on the top of the wall. The base of the ladder makes an angle of 30° with the ground.

(i) Find the height of the wall.

(ii) If the ladder is moved so that its base moves 3 m towards the wall, find α, the angle which the ladder now makes with the ground, correct to two decimal places.

(iii) Find the length of the ladder that then protrudes above the wall. Give your answer correct to two decimal places.

[7 marks]



Q5

(a) Differentiate the function $g(x) = x^2 + 2x + 3$ from first principles.

[5 marks]

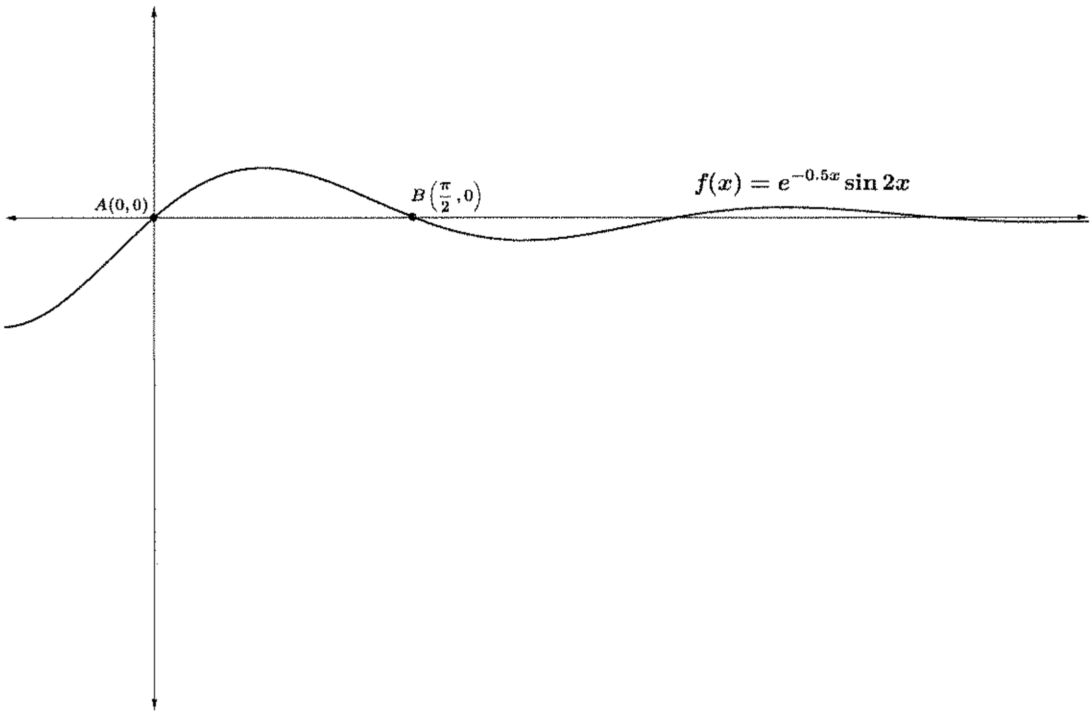
(b) The graph of the function $f(x) = e^{-0.5x} \sin 2x$ is shown below.

(i) Find an expression for $f'(x)$.

(ii) Hence find the equation of the tangent to the graph of the function $f(x)$ at the point $(0, 0)$.

(iii) Find the slope of the tangent to the graph of the function $f(x)$ at the point $\left(\frac{\pi}{2}, 0\right)$.

[7 marks]



Question 5 continued overleaf

Question 5 continued

(c) A curve $f(x)$ passes through the point $(0, 4)$ and the slope of the tangent to the curve $f(x)$ at any point is given by $f'(x) = 2\cos x + \sin x$.

Find an expression for $f(x)$.

[3 marks]

(d) The diagram below shows part of the curve $f(x) = x^3 - 2x^2 - x + 2$.

Determine the value of the shaded area.

[5 marks]