CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2011

Module Title: Technological Mathematics 201

Module Code: MATH 6040

School: Mechanical and Process Engineering
School of Building, Civil and Environmental Engineering
Manufacturing, Biomedical and Facilities Engineering

Programme Title: Bachelor of Engineering in Civil Engineering
Bachelor of Engineering in Mechanical Engineering
Bachelor of Engineering in Biomedical Engineering
Bachelor of Engineering in Building Services Engineering
Bachelor of Engineering in Sustainable Energy Technology

Programme Code: ESENT_8_Y2
EMECH_7_Y2
EBIME_7_Y2
EBSEN_7_Y2
CCIVL_7_Y2

External Examiner(s): Dr. P. Kirwan
Internal Examiner(s): Dr. M. Lishchynska, Dr. J. O'Donovan, Ms. C. Palmer

Instructions: **Answer Question 1 (compulsory) (worth 40 marks)**
AND one question from Section B (worth 30 marks)
AND one question from Section C (worth 30 marks)

Duration: 2 hours

Sitting: Semester 2 2011

Requirements for this examination: Mathematical Tables

Note to Candidates: Please check the **Programme Title** and the **Module Title** to ensure that you have received the correct examination.

If in doubt please contact an Invigilator.

Section A

Q1

(a) Given the ellipse with the equation

$$2x^2 + y^2 = 24$$

(i) Find a general expression for $\frac{dy}{dx}$.
(ii) Find the two points on the ellipse with $x = 2$.
(iii) Find the value of $\frac{dy}{dx}$ at each of these points.
(iv) Find the equation of the tangent to the ellipse at each of these points.

[5 marks]

(b) If $x = 4(t - \sin t)$, and $y = 4(1 - \cos t)$ determine $\frac{d^2y}{dx^2}$.

[5 marks]

(c) Evaluate $\int \ln x \, dx$.

[5 marks]

(d) The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the x-axis is revolved about the x-axis to generate a solid. Find the volume of this solid.

[5 marks]

(e) If

$$\begin{aligned} \mathbf{a} &= 3i + 2j - 2k \\ \mathbf{b} &= -i - 3j + 5k \\ \mathbf{c} &= i + j - 4k \end{aligned}$$

find
(i) $|\mathbf{a}|$
(ii) $|\mathbf{a} - \mathbf{b}|$
(iii) $\mathbf{a}.(\mathbf{b} \times \mathbf{c})$
(iv) $(\mathbf{a} - \mathbf{b}) \times \mathbf{c}$

[10 marks]

(f) Given

$$\mathbf{C} = \begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ -\frac{1}{\sqrt{5}} & k \end{pmatrix}$$

and $\mathbf{C}^T\mathbf{C} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix, find the value of k.

[5 marks]

(g) If

$$\mathbf{A} = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix},$$

find the values of x and y given that $\mathbf{AB} = \mathbf{BA}$.

[5 marks]

Section B

Q2

(a) For the matrices

$$\mathbf{A} = \begin{pmatrix} 7 & 5 \\ 4 & 3 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}; \quad \mathbf{C} = \begin{pmatrix} 1 & 0 & 3 \\ -1 & -2 & 0 \end{pmatrix}$$

(i) Determine, if defined, the following:

- $\mathbf{A} + \mathbf{B}$
- $(\mathbf{A} + \mathbf{B})^T$
- $\mathbf{B} + \mathbf{C}$
- $\mathbf{CA}$
- $\mathbf{AC}$

(ii) Determine the matrix $\mathbf{X}$ such that $\mathbf{AX} = \mathbf{B}$.

[11 marks]

(b) Determine for what values of λ the following matrix has no inverse:

$$\begin{pmatrix} \lambda - 1 & 3 \\ 2 & 3\lambda \end{pmatrix}$$

[3 marks]

(c) Use the method of matrix inversion to solve the following system of simultaneous equations.

$$\begin{aligned} 3x + 5y - z &= -3 \\ 2x + y - 3z &= -9 \\ x + 3y + 2z &= 7 \end{aligned}$$

Confirm, using Cramer's rule, your answer for the variable z.

[16 marks]

Q3

(a) Let $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j}$ and $\mathbf{w} = -2\mathbf{i} + \mathbf{j}$

(i) Sketch $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{v} + \mathbf{w}$ as position vectors and show how the parallelogram law is used to calculate $\mathbf{v} + \mathbf{w}$.

(ii) Write down $\mathbf{v} + \mathbf{w}$ in terms of its Cartesian components.

(iii) Find $|\mathbf{v} + \mathbf{w}|$.

(iv) Determine the angle made by the vector $\mathbf{v} + \mathbf{w}$ with the positive $x-$axis.

[9 marks]

(b) **(i)** Given the points $0\,(0,0,0)$, $A\,(2,-1,0)$, $B\,(7,-4,3)$ and $C\,(13,-7,3)$ let

$$\begin{aligned} \mathbf{a} &= \overrightarrow{0A} \\ \mathbf{b} &= \overrightarrow{0B} \\ \mathbf{c} &= \overrightarrow{0C} \end{aligned}$$

show that the vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are coplanar.

(ii) Find a unit vector perpendicular to this plane.

[8 marks]

(c) Find the work done by the force $\mathbf{F} = 5\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$ in moving an object from the point $A(4,-5,7)$ to the point $B\,(7,-4,5)$. Assume that force is measured in Newtons and distance in metres.

[4 marks]

(d) A force $\mathbf{F} = -3\mathbf{i} + 7\mathbf{j} + \mathbf{k}$ acts at the point $A\,(-2,3,1)$ find the moment of the force about the origin. Assume that force is measured in Newtons and distance in metres.

[5 marks]

(e) For what values of a are the vectors $-2\mathbf{i} + a\mathbf{j} - 3\mathbf{k}$ and $5\mathbf{i} - 2a\mathbf{j} + 4a\mathbf{k}$ perpendicular?

[4 marks]

Section C

Q4

(a) Determine the equation of the tangent line drawn to the curve $x = 3t$, $y = 3\sqrt{t}$ at the point $(3, 3)$.

[7 marks]

(b) Given $7x^2 + 3x^3y - 4y^2 = 0$ determine $\frac{dy}{dx}$ and hence find the slope of the tangent to the curve at the point $(1, -1)$.

[7 marks]

(c) The shear stress q in a shaft of diameter D under torque T is $q = \frac{16T}{\pi D^3}$. Using differentials, find the approximate percentage error in the calculated value of q if T is measured 3% too large and D is measured 2% too small.

[8 marks]

(d) Show that if $z = e^{-t}\sin\theta$ then

$$\frac{\partial^2 z}{\partial\theta\partial t} = \frac{\partial^2 z}{\partial t\partial\theta}.$$

[8 marks]

Q5

(a) Determine the following integral $\int 5xe^{2x}\,dx$.

[4 marks]

(b) Find the following integral

$$\int_{-3}^{-1} \frac{1}{t^2+6t+13}\,dt.$$

[4 marks]

(c) The force F (Newtons) acting on an object is given by $F = \frac{5}{\sqrt{4-x^2}}$, where x is the displacement of the body in metres. Find the work done in moving the object from $x = 0$ to $x = 1$ metre.

[6 marks]

(d) Figure 1 shows a graph of $y = (x-5)^2$. Determine

(i) the area bounded by the curve $y = (x-5)^2$, the x-axis and the ordinates $x = 0$ and $x = 5$,

(ii) the coordinates of the centroid of this area,

(iii) the volume of the solid produced by revolving $y = (x-5)^2$ around the x-axis and between the limits $x = 0$ and $x = 5$,

(iv) the co-ordinates of the centre of gravity of this solid.

[16 marks]

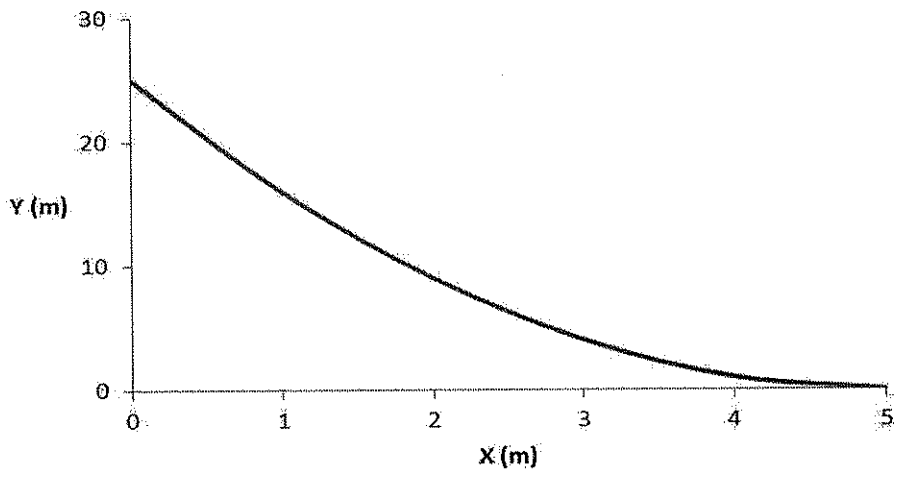

Figure 1: Graph of $y = (x-5)^2$

—Required Formulae—

Centroid $\bar{x} = \dfrac{\int xy\ dx}{\int y\ dx}$; $\bar{y} = \dfrac{\frac{1}{2}\int y^2\ dx}{\int y\ dx}$

Volume $V = \pi \int y^2\ dx$

Centre of Gravity $\bar{x} = \dfrac{\int xy^2\ dx}{\int y^2\ dx}$; $\bar{y} = 0$

Work done by Variable Force

$$W = \int F\ ds$$

$$W = \int P\ dV$$

$$W = \int T\ d\theta$$