CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2010/11

Module Title: Calculus 1 for Computing

Module Code: **MATH6003**

School: Science and Informatics

Programme Title:
Bachelor of Science (Honours) in Environmental Science & Sustainable Technology
Bachelor of Science (Honours) in Instrument Engineering
Bachelor of Science (Honours) in Software Development
Bachelor of Science (Honours) in Software Development and Computer Networking

Programme Code:
CR_SESST_8_Y1
CR_SINEN_8_Y1
CR_KSDEV_8_Y1
CR_KDNET_8_Y1

External Examiner(s): **Dr. Paul Robinson**

Internal Examiner(s): **Dr Seán Lacey, Dr Áine Ní Shé**

Instructions:
Answer Question 1 (compulsory – 40 marks) and TWO other questions (30 marks each)

Duration: **2 hours**

Sitting: Summer 2011

Requirements for this examination: Formulae & Tables Booklet (State Examinations Commission)

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

1. (a) Given the functions

$$y_1(t) = 50e^{-100t}, \qquad y_2(t) = 40(1 - e^{-100t}).$$

where t is time in seconds:

i. State the initial and final values of each of the functions $y_1(t)$ and $y_2(t)$.

[2.5 marks]

ii. Calculate the value of $y_2(0.015)$.

[1.5 marks]

iii. Determine the time t at which $y_1(t) = y_2(t)$.

[4 marks]

(b) i. State the amplitude, period and frequency of the waveforms $v_1(t)$, $v_2(t)$ below.

[2.5 marks]

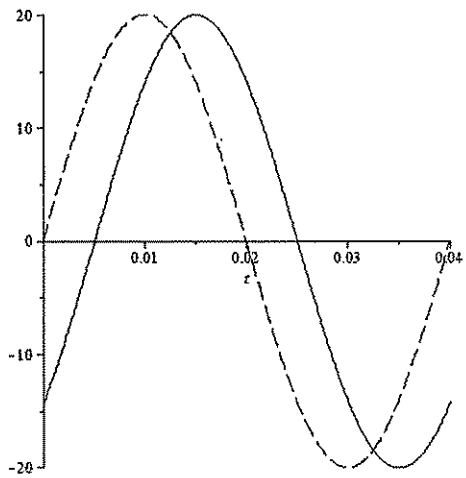

Figure 1: Sketch of the waveforms $v_1(t)$, $v_2(t)$.

ii. Hence, determine an expression for the waveform $v_1(t)$.

[1.5 marks]

iii. State the phase time of the waveform $v_2(t)$. Hence, calculate its phase angle.

[2.5 marks]

iv. Hence, determine an expression for the waveform $v_2(t)$.

[1 mark]

v. Hence, calculate the value of $v_2(0.01)$.

[1.5 marks]

(c) Differentiate, from first principles, the function

$$f(x) = x^2 - 5x - 6.$$

[5 marks]

(d) For the function $v(t) = 10\cos 50\pi t$, determine $v'(0.13)$.

[4 marks]

(e) i. Find the slope of the tangent line to the curve

$$y = \frac{2}{x},$$

at the point where $x = 2$.

[2 marks]

ii. Hence, determine the equation of the tangent line.

[2 marks]

(f) Calculate the value of the integral

$$\int_{-1}^{2} 4x^3 + 8x + 1 \, \mathrm{d}x.$$

[5 marks]

(g) Find the mean value of the function

$$f(t) = 1 - \mathrm{e}^{-t},$$

over the interval $[0, 2]$.

[5 marks]

2. (a) Differentiate each of the following with respect to the variable:

i. $\ln(x^2+1)$,

[2.5 marks]

ii. $e^{\sin 2x}$,

[3.5 marks]

iii. $e^{-0.4t}\cos 2t$,

[5 marks]

iv. $\frac{2w+1}{w^2}$.

[4 marks]

(b) The distance (measured in metres) that a particle, dropped into a viscous fluid, travels in t seconds is given by

$$f(t) = 80(1 - e^{-0.25t}).$$

Determine

i. the instantaneous speed of the particle at the instant when $t = 3$ seconds;

[2.5 marks]

ii. the time when the speed of the particle is 25 metres per second.

[2.5 marks]

(c) i. Find the coordinates of the maximum and minimum turning points of the function

$$f(x) = x^3 - \frac{3x^2}{2} - 6x + 3.$$

[7 marks]

ii. Hence, sketch the graph of the function $f(x)$.

[3 marks]

3. (a) Evaluate each of the following integrals:

i. $\int \frac{x^2+x+1}{x}\,\mathrm{d}x$,

[4 marks]

ii. $\int \cos x \cdot \mathrm{e}^{\sin x}\,\mathrm{d}x$,

[4 marks]

iii. $\int_0^{0.5}(2t+1)^5\,\mathrm{d}t$,

[5 marks]

iv. $\int_0^{\pi/6}\cos^2 3\theta\,\mathrm{d}\theta$.

[5 marks]

(b) Figure 2 below shows the graph of the function $f(x) = \frac{1}{x^2+4}$. Find the mean value of this function over the interval $[2, 5]$.

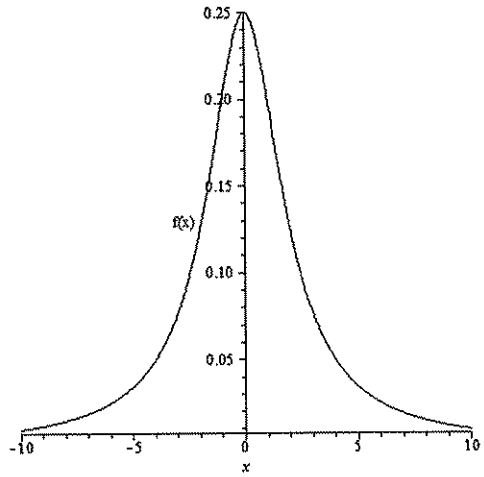

Figure 2: Graph of the function $f(x) = \frac{1}{x^2+4}$.

[5 marks]

(c) Solve the differential equation $f''(t) = \mathrm{e}^{-2t}$ given that $f(0) = 0$ and $f'(0) = 0$.

[7 marks]

4. (a) i. Show that the equation $x - 3\ln x = 0$ has a root between $x = 1$ and $x = 2$.

[2 marks]

ii. Taking $x = 1.8$ as an initial approximation, use two iterations of the Newton-Raphson method to approximate the value of this root correct to two decimal places.

[8 marks]

(b) A projectile is fired vertically upwards and its height, h, at time t, is given by

$$h = h(t) = 10t - 4.9t^2.$$

Find

i. the time at which the particle reaches its maximum height ,

[5 marks]

ii. the maximum height reached by the particle.

[2 marks]

(c) i. Sketch, on the same set of axes, graphs of the functions

$$f(x) = 3 - x^2, \qquad g(x) = x + 1,$$

over the interval $[-3, 3]$.

[5 marks]

ii. Confirm algebraically that the graphs of $f(x)$ and $g(x)$ intersect at the points $x = -2$, $x = 1$.

[3 marks]

iii. Hence, calculate the area bounded between these two curves.

[5 marks]