CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2010/11

Module Title: Technological Mathematics 2

Module Code: MATH 6015

School: School of Building and Civil Engineering

Programme Title:
Bachelor of Engineering in Civil Engineering – Year 1

Programme Code: **CCIVL_7_Y1**

External Examiner(s): Dr. P. Kirwan.
Internal Examiner(s): Ms H. Lordan.

Instructions: **Answer QUESTION 1 (compulsory - 40 marks) and TWO other questions (30 marks each)**

Duration: 2 Hours

Sitting: Summer 2011

Requirements for this examination: Mathematical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.

1 (a) Differentiate from first principles: $f(x) = 3x - 5x^2$

(5marks)

(b) Find the derivative of $y = \frac{2}{5}x^3 - \frac{4}{x^2} + 7\sqrt{x^5} + 10$

(5marks)

(c) If $f(t) = 3\sin(4t) - 2\cos(3t)$ determine $f^{'}(t)$ and $f^{''}(t)$.

(5marks)

(d) Evaluate $\frac{dy}{dx}$ when $x = 1$ given $y = 3e^{4x} + 8\ln(5x)$

(5marks)

(e) Determine $\int (2 + \frac{5}{7}x - 6x^2)dx$

(5marks)

(f) Evaluate $\int_0^4 \frac{3}{\sqrt{25 - x^2}}dx$

(5marks)

(g) The acceleration of an object is given by $a = 20e^{-0.5t}$ m s^{-2} where t is the time in seconds. Derive an expression for velocity and an expression for displacement if both the initial velocity and initial displacement are zero.

(5marks)

(h) Determine $\int_1^3 (3x - 2)^5 dx$

(5marks)

2 (a) Differentiate by rule:

(i) $y = \dfrac{2x}{x^2 + 1}$

(ii) $y = \sqrt{4x^2 + 3x - 5e^{2x}}$

(iii) $y = x^3 \cos(3x)$

(12 marks)

(b) An object falls from a very tall building. The distance it falls in a time t seconds is given by $x = \frac{1}{2}gt^2$ where $g = 9.8\,\mathrm{m\,s^{-2}}$. Determine the velocity and acceleration of the object after it has fallen for 2.5 seconds.

(8 marks)

(c) Find the co-ordinates of the turning points on the curve $y = 2x^3 - 15x^2 + 24x + 2$ and determine whether each is a maximum point, minimum point or point of inflection.

(10 marks)

3 (a) Determine each of the following:

(i) $\int_{0}^{3}(x^2 - 2e^{3x} + \sqrt{x})dx$

(ii) $\int \frac{3x+5}{(x+3)(x-1)}dx$

(iii) $\int_{2}^{4}\frac{x^2 - 1}{\sqrt{x^3 - 3x - 1}}dx$

(18 marks)

(b) Find the points of intersection of the curve $y = 12x - x^2$ and the line $y = 8x$. Sketch the curve and the line showing the enclosed area. Use integration to find this area.

(12 marks)

4 (a) A closed tank has a square base of side x mm. The volume of the tank is $8\,\mathrm{m}^3$. Show that the total surface area is $A = 2x^2 + \frac{32}{x}\,\mathrm{mm}^2$. Use differentiation to find the dimensions of this tank to minimise the surface area.

(10 marks)

(b) Determine (i) the mean value of, and
(ii) the root mean square (r.m.s.) of
$y = 2x^2 + 5$ between $x = 1$ and $x = 3$.

(10 marks)

(c) Given that $\frac{dr}{dt} = \frac{-50}{t^2}$ and that $r = 2$ when $t = 10$, find the value of r when $t = \frac{1}{2}$.

(10 marks)

Mean Value

$$\frac{1}{b-a}\int_a^b f(x)dx$$

Root Mean Value

$$RMS = \sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2 dx}$$