CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2010/11

Module Title: Technological Mathematics 2 & Maple

Module Code: **MATH 6019**

School: Computing and Mathematics

Programme Title: Bachelor of Science in Applied Physics and Instrumentation – Year 1

Programme Code: **SPHYS_7_Y1**

External Examiner(s): **Dr. Padraig Kirwan.**
Internal Examiner(s): **Mr. P. Ahern.**

Instructions: **Answer Question 1 - Compulsory (40 Marks) and TWO other questions (30 Marks each)**

Duration: 2 Hours

Sitting: Summer 2011

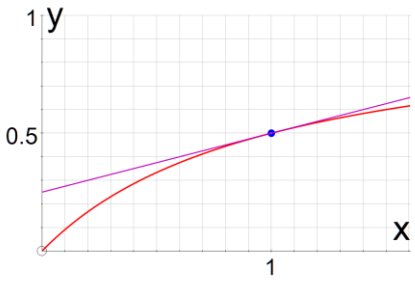
Requirements for this examination: Formulae and tables are provided with this paper.

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.
If in doubt please contact an Invigilator.

1 **COMPULSORY** [40 marks]

(a) Differentiate $y = 3x^2 - 4x + 6$ from first principles. [5 marks]

(b) Find the equation of the tangent to the curve $y = \dfrac{x}{1+x}$ at the point (1, 0.5).



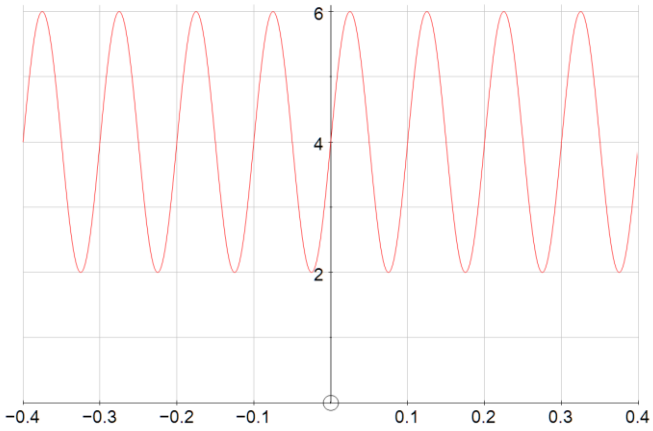
[5 marks]

(c) The temperature $\theta(t)$ in ^{0}C of a body at time t s is given by $\theta(t) = 20 + 60e^{-0.1t}$.

Find the rate at which the temperature is changing when $t = 10$ s. [5 marks]

(d) The displacement x m of a particle at time t s is described by $x = 4 + 2\sin(20\pi t)$.

Find the velocity of the particle at $t = 0$. [5 marks]

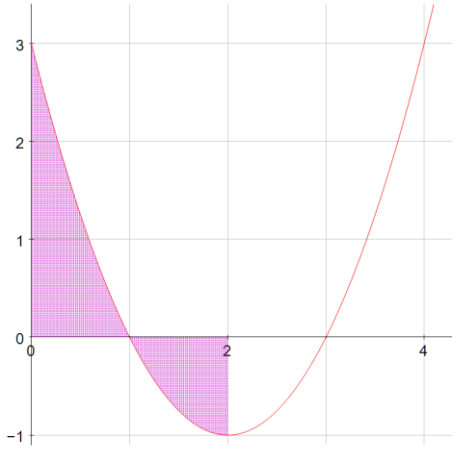


(e) Evaluate the definite integral $\int_0^1 \dfrac{x}{1+x^2}\,dx$. [5 marks]

(f) The acceleration a ms^{-2} of a particle at time t s is given by $a = 10 - 0.4t$. Find the velocity of the particle at any time t, given that the initial velocity is zero.

[5 marks]

(g) Find the area enclosed by the curve $y = 3 - 4x + x^2$ and the x axis between $x = 0$ and $x = 2$.



[5 marks]

(h) The current $i(t)$ mA in a circuit at time t seconds is given by $i(t) = 20(1 - e^{-0.4t})$.

Find the mean current between $t = 0$s and $t = 4$s. [5 marks]

2. [30 marks]

(a) Differentiate each of the following with respect to x:

(i) $y = 1.6 + x - \frac{2}{x} + \frac{1}{\sqrt{x}}$

(ii) $y = \sqrt{a^2 - x^2}$

(iii) $y = e^{2x}\sin(4x)$

(iv) $y = \frac{\cos\left(\frac{x}{2}\right)}{x+1}$ [14 marks]

(b) The displacement s m of a body at time t s is given by

$$s = 0.5 + 1.2t - 0.02t^3$$

Find (i) the velocity of the body at $t = 0$ and

(ii) the time at which the velocity is zero. [8 marks]

(c) Given the function $y = x^2 + \frac{1}{x^2}$,

(i) find the slope of the tangent to the curve at the point at which $x = 2$.

(ii) find the points on the curve at which the slope is zero. [8 marks]

3. [30 marks]

(a) Evaluate each of the following integrals:

(i) $\int\left(1-\frac{1}{t^2}+\frac{2}{t^4}\right)dt$

(ii) $\int_0^1 \frac{10x}{9+x^2}\,dx$

(iii) $\int_0^{0.25}\left(4\sin(4\pi t)-3\cos(2\pi t)\right)dt$ [12 marks]

(b) A specially designed template is defined as the area enclosed by the curve $y=0.5x^2$ and the line $y=0.2x+0.3$ between $x=0$ and $x=1$. Find this area.

[8 marks]

(c) Water flows into a parallel sided tank at a constant rate and out through a valve which acts as a linear flow resistance. Under these circumstances the differential equation describing the level h m of water at any time t seconds is described by the differential equation

$$20\frac{dh}{dt}+h=0.5$$

(i) Solve this equation subject to the initial condition $h=0$ when $t=0$.

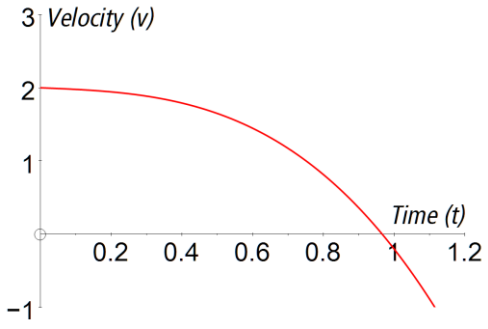
(ii) State the approximate time taken for the level to reach its steady state value.

[10 marks]

4. [30 marks]

(a) The force F (kN) acting on a body is given by $F = 4x(1-0.001x^2)$ where x is the displacement of the object in cm. Find the work done in moving the body from $x = 0$ cm to $x = 20$ cm. Find also the mean value of the force over this interval. State your units. [10 marks]

(b) The velocity v ms^{-1} of a body at time t s is given by $v = 2 - 0.2t - 2t^3$. Use the Newton Raphson method to find the time at which the velocity is zero correct to 3 decimal places.



[10 marks]

(c) The voltage $v(t)$ mV in a circuit is described by the differential equation

$$v(t) = 20t\,e^{-0.5t}$$

where t is the time elapsed in seconds.

Find (i) the rate at which the voltage is changing when $t = 0$ and

(ii) the maximum voltage. [10 marks]