

**CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ**

Semester 2 Examinations 2010/11

Module Title: Calculus & Statistics 2

Module Code: MATH 6029

School: Science

Programme Title: Bachelor of Science in Analytical & Pharmaceutical Chemistry – Year 2

Programme Code: SCHEM_7_Y2

External Examiner(s): Dr. P. Kirwan

Internal Examiner(s): Ms. F. Wood

**Instructions: Answer question 1 worth 40 marks.
Answer ONE question from Section A and ONE question from Section B.
Total marks = 100**

Duration: 2 Hours

Sitting: Summer 2011

Requirements for this examination: Mathematics Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.
If in doubt please contact an Invigilator.

Q.1(a) A curve is described in parametric form by $x = t^2$, $y = \frac{1}{t^2}$. Find $\frac{dy}{dx}$ for the function and hence find the equation of the tangent to the curve at $t = 3$.

(8 marks)

(b) The modulus of rigidity G is defined as $G = \frac{R^4 \theta}{L}$ where R = radius, θ = angle of twist and L = length. Determine the approximate percentage change in G when R increases by 1% and L increases by 2% . θ remains constant.

(8 marks)

(c) Determine

$$\int x^3 \ln(x) dx$$

(8 marks)

(d) It is found that 28% of blood samples taken from people suffering muscle cramp have 'high' levels of potassium. Blood samples are taken from 12 randomly selected patients. Find the probability that

- (i) three samples
- (ii) at least two samples have 'high' potassium levels.

(8 marks)

(e) A survey of a college computer system helpdesk was carried out. A random sample of 180 requests to the helpdesk showed that 54 of them were taking 15 minutes or more to process.

Find a 95% confidence interval for the proportion of all requests taking at least 15 minutes to process. Interpret your result.

(8 marks)

Section A

Q.2(a) Show that the point $(2, -1)$ lies on the curve $3x^2 - 4y^3 = 24 - x^3y^2$

Find $\frac{dy}{dx}$ for the function and find the slope of the tangent to the curve at this point.

Find where the function crosses the x -axis.

(12 marks)

(b) If

$$V = \ln(x^2 + y^2)$$

find partial derivatives $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial y}$. Express $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y}$ in its simplest form.

Find $\frac{\partial^2 V}{\partial x \partial y}$.

(8 marks)

(c) The blood pressure P (mm Hg) of a woman varies with age t (years) and level of physical exercise E according to the equation

$$P = 217 - 0.85t - \frac{7E}{t}$$

- (i) What is the blood pressure of a 45 year old woman whose exercise level is 100?
- (ii) Determine her rate of change of blood pressure with respect to age at this point.
- (iii) Determine her rate of change of blood pressure with respect to exercise at this point.

(10 marks)

Q.3(a) Evaluate the following integral:

$$\int_4^6 4x\sqrt{x^2 - 6} dx$$

(9 marks)

(b) Calculate the root mean square (r.m.s.) of the function $y = x + \frac{5}{x^2}$ in the region $x = 2$ to $x = 3$.

(12 marks)

(c) A gas expands according to the law $PV = C$.

(i) Find the constant C if initially the pressure is 200 kNm^{-2} when the volume is 2m^3 .

(ii) Find the work done by the gas if it expands to twice its initial volume.

$$\text{Work} = \int P dV$$

(9 marks)

Section B

Q.4(a) The development of Free Fatty Acids (FFA) in herring stored at 6°C was studied during a 20-day period. The results are tabulated as follows:

No. of days storage (x)	1	7	12	15	20
FFA (%) (y)	3.2	12.8	16.4	28.0	37.1

- (i) Plot the data on a scatter diagram. (5 marks)
- (ii) Calculate the equation of the regression line of % FFA on storage and fit this line to your graph in (i). (12 marks)
- (iii) Calculate a coefficient of correlation for the data. Interpret your result. (5 marks)
- (b) A company providing maintenance for laboratory equipment receives on average 18 callouts per 5-day week. Calculate the probability that on any given day the centre will receive
- (i) less than three
- (ii) more than three callouts. (Assume a Poisson Distribution) (8 marks)

- Q.5(a) The duration of telephone calls to a company has been found to be approximately normally distributed with a mean duration of 4.8 minutes and a standard deviation of 1.2 minutes. Calculate the proportion of calls that have a duration of
- more than six minutes
 - less than two minutes
 - between 2 and 5 minutes.

(11 marks)

- (b) The following table shows the protein content in a sample of seven brands of baking flour:

Brand	A	B	C	D	E	F	G
g /100g	10.0	10.4	9.8	11.2	9.3	12.1	10.6

- Calculate the mean and standard deviation of the data.
- Calculate a 95% confidence interval for the population mean and interpret your result.

(10 marks)

- (c) A chemical plant is producing cartons of a substance and the mean carton weight is supposed to be 1kg. The production manager suspects that the filling machine is underweighing. A random sample of 30 cartons shows a mean of 0.989kg and a standard deviation of 0.028kg. Test the manager's suspicion at the 5% significance level and at the 1% significance level and interpret your results.

(9 marks)

Statistical Formulae

1. Descriptive Statistics

$$\text{Mean: } \bar{x} = \frac{\sum x_i}{n} \qquad \text{Standard Deviation: } s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

2. Regression and Correlation

$$y' = a + bx$$

$$\begin{aligned} \sum y &= na + b\sum x \\ \sum xy &= a\sum x + b\sum x^2 \end{aligned} \qquad \bar{y} = a + b\bar{x}$$

$$a = \frac{\sum y - b\sum x}{n} \qquad b = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$$

Coefficient of Correlation:

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \cdot \sqrt{n\sum y^2 - (\sum y)^2}}$$

3. Probability Distributions

(i) Binomial Distribution

$$P(r,n) = {}^n C_r p^r q^{n-r}$$

(ii) Poisson Distribution

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}$$

(iii) Normal Distribution

$$\text{Standard Units } z = \frac{x - \bar{x}}{s} = \frac{x - \mu}{\sigma}$$

4. Sampling

$$\text{Standard Error of the mean} = \frac{s}{\sqrt{n}}$$

$$\text{Standard Error of the proportion} = \sqrt{\frac{pq}{n}}$$

$$Z\text{-score} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$