CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2010/11

Module Title: Mathematics for Science 2.2 with Maple

Module Code :	**MATH 6038**
School :	School of Science and Informatics
Programme Title :	Bachelor of Science in Applied Physics and Instrumentation – Year 2 Higher Certificate in Industrial Measurement and Control – Year 2
Programme Code :	SPHYS_7_Y2 SIMCT_6_Y2
External Examiner :	Dr. P. Robinson
Internal Examiners :	Dr. M. Brennan
Instructions :	Answer Q1(compulsory) and 2 other questions.
Duration :	2 Hours
Sitting :	Summer 2011
Requirements :	Mathematical Tables

Q1. (a) Determine A^{-1} where $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & -4 \\ 2 & 4 & -3 \end{bmatrix}$.

(10 marks)

(b) A component is defective if it is oversized. A sample of 460 components produced by a machine have a mean size of 7.2 cm and a standard deviation of 0.12 cm. The maximum size of an acceptable component is 7.46 cm. Assume a *normal distribution.*

(i) Determine how many components are defective.

(ii) Find how many components are acceptable.

(8 marks)

(c) Calculators in the CIT shop come in three varieties, *Sharp*, *Casio* or *TI.* Of all such calculators, 55% are *Sharp*, 35% *Casio*, and 10% *TI.* Further it is known that 2% of all *Sharp* calculators are defective, 1% of *Casio* are defective, and 3% of *TI* are defective.

(i) Represent this information in a tree diagram.

(ii) Find the probability that a calculator chosen at random is a *Sharp* or a *TI* calculator.

(iii) Find the probability that a calculator chosen at random is defective.

(iv) A certain pocket calculator is found to be defective in the CIT shop. Find the probability that this defective calculator is a *Sharp.*

(12 marks)

(d) The number of misprints in a book satifies a *Poisson distribution.* Suppose 300 misprints are distributed randomly throughout a book of 500 pages. Find the probability that a given page contains

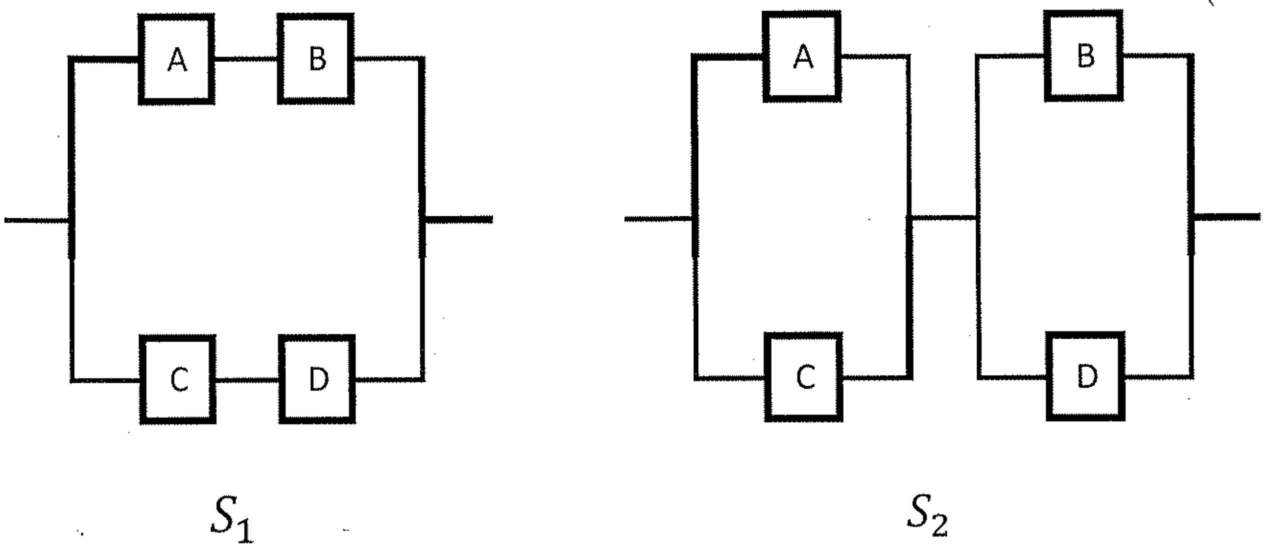
(i) exactly 2 misprints,

(ii) 2 or more misprints.

(8 marks)

Q1. (e) A *Reliability Block Diagram* is given for two systems S_1 and S_2. Determine $P(S_1)$ and $P(S_2)$ and hence identify the most reliable system where the reliabilities P(A)=0.8, P(B)=0.9 P(C)=0.7 and P(D)=0.6. Carefully show all steps and intermediate calculations.

(12 marks)



Q2. (a) (i) Apply **only** the *Gauss-Jordan Method* to solve the system of linear equations

$$\begin{aligned} x + y + 2z &= -2 \\ 3x + 6y - 5z &= 8 \\ 2x + 4y - 3z &= 10. \end{aligned}$$

(ii) Verify z using *Cramer's Rule*.

(14 marks)

(b) Wires manufactured for use in a certain computer system are specificed to have resistances between 0.12 and 0.14 ohm. The actual measured resistances of the wires produced by Company A have a normal probability distribution with a mean of 0.13 ohm and a standard deviation of 0.005 ohm.

(i) What is the probability that a randomly selected wire from Company A's production will meet the specifications?

(ii) If four such wires are used in the system and all are selected from Company A, what is the probability that all four will meet the specifications?

(11 marks)

Q3.(a) A sample of 50 ball bearings was taken from the production of machine A and their diameters (in cm) where measured to give the following distribution.

x	0.60-0.62	0.63-0.65	0.66-0.68	0.69-0.71	0.72-0.74	0.75-0.77	0.78-0.80
f	2	3	10	18	14	2	1

(i) Find the *mode* for the above distribution.

(ii) Use the *assumed mean method* to determine the mean and standard deviation.

(iii) A second sample of 100 ball bearings was taken from machine B, giving a mean diameter of $0.65cm$ with standard deviation of $0.03cm$. Compute the *coefficient of variation* for each machine. Which machine has the greater variation?

(18 marks)

(b) Suppose a large lot contains 10% defective fuses. Assuming a binomial distribution find the probability that in a sample of eight fuses, either three or four fuses are defective.

(7 marks)

Q4. In order to monitor the quality of a production run of aluminium bolts, 8 samples, each containing 4 components, are taken at random and their diameter lengths are measured correct to the nearest 0.1mm and tabulated as follows:

Sample	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	89.4	92.2	87.7	89.2	81.2	91.7	81.8	90.2
	89.9	90.1	85.1	89.4	82.0	89.9	81.8	90.2
	91.9	91.3	82.3	90.8	82.1	88.2	90.3	87.3
	90.8	91.4	90.9	89.8	81.3	80.2	81.9	89.3
Means $\overline{x}_i$	90.50	91.25	$\overline{x}_3$	89.8	$\overline{x}_5$	$\overline{x}_6$	$\overline{x}_7$	89.25
Ranges w_i	2.5	2.1	w_3	1.6	w_5	w_6	w_7	2.9

(a) Use sample 6 to set up 95% and 99% confidence intervals for the population mean. Comment briefly on your answers.

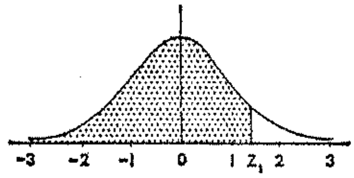
(10 Marks)

(b) Calculate the remaining sample means $\overline{x}_i$ and ranges w_i. Find the grand mean $\overline{\overline{x}}$ and the associated *outer* and *inner control limits*. Hence set up a control chart for the sample means. State, giving reasons, whether or not the process is under control.

(15 Marks)

Normal Distribution Tables

Area under the Normal Curve $P(z \le z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{z^2}{2}} dz$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997

Statistical Formulae

Sample mean:

$$\bar{x} = \frac{\Sigma f x}{\Sigma f} = A + c\frac{\Sigma f d}{\Sigma f}, \qquad \text{where } d = \frac{x - A}{c}$$

Population mean:

$$\mu = \frac{\Sigma f x}{\Sigma f} = A + c\frac{\Sigma f d}{\Sigma f}, \qquad \text{where } d = \frac{x - A}{c}$$

Sample standard deviation:

$$s = \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f - 1}} = c\sqrt{\frac{\Sigma f d^2}{\Sigma f - 1} - \frac{\Sigma f}{\Sigma f - 1}\left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

Population standard deviation:

$$\sigma = \sqrt{\frac{\Sigma f(x - \mu)^2}{\Sigma f}} = c\sqrt{\frac{\Sigma f d^2}{\Sigma f} - \left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

Coefficient of Variation

$$CoV = \frac{s}{\bar{x}} \times 100$$

Binomial distribution:

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Poisson distribution:

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!} = e^{-\lambda} \cdot \frac{\lambda^r}{r!}$$

Normal distribution:

$$z = \frac{x - \mu}{\sigma}$$

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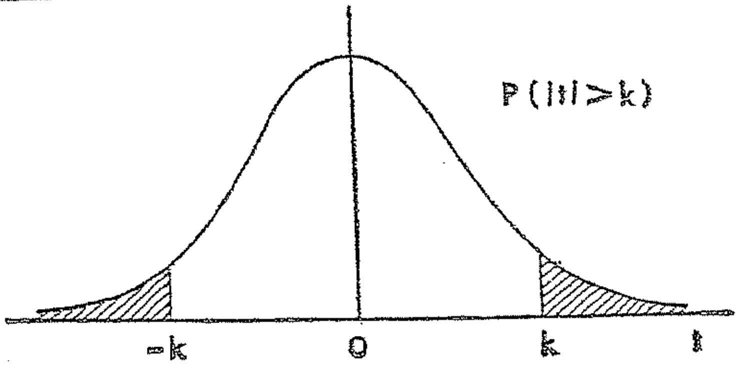
n − 1	20	10	5	2	1	0·2
1	3·078	6·314	12·706	31·821	63·657	318·310
2	1·886	2·920	4·303	6·965	9·925	22·327
3	1·638	2·353	3·182	4·541	5·841	10·215
4	1·533	2·132	2·776	3·747	4·604	7·173
5	1·476	2·015	2·571	3·365	4·032	5·893
6	1·440	1·943	2·447	3·143	3·707	5·208
7	1·415	1·895	2·365	2·998	3·499	4·785
8	1·397	1·860	2·306	2·896	3·355	4·501
9	1·383	1·833	2·262	2·821	3·250	4·297
10	1·372	1·812	2·228	2·764	3·169	4·144
11	1·363	1·796	2·201	2·718	3·106	4·025
12	1·356	1·782	2·179	2·681	3·055	3·930
13	1·350	1·771	2·160	2·650	3·012	3·852
14	1·345	1·761	2·145	2·624	2·977	3·787
15	1·341	1·753	2·131	2·602	2·947	3·733
16	1·337	1·746	2·120	2·583	2·921	3·686
17	1·333	1·740	2·110	2·567	2·898	3·646
18	1·330	1·734	2·101	2·552	2·878	3·610
19	1·328	1·729	2·093	2·539	2·861	3·579
20	1·325	1·725	2·086	2·528	2·845	3·552
21	1·323	1·721	2·080	2·518	2·831	3·527
22	1·321	1·717	2·074	2·508	2·819	3·505
23	1·319	1·714	2·069	2·500	2·807	3·485
24	1·318	1·711	2·064	2·492	2·797	3·467
25	1·316	1·708	2·060	2·485	2·787	3·450
26	1·315	1·706	2·056	2·479	2·779	3·435
27	1·314	1·703	2·052	2·473	2·771	3·421
28	1·313	1·701	2·048	2·467	2·763	3·408
29	1·311	1·699	2·045	2·462	2·756	3·396
30	1·310	1·697	2·042	2·457	2·750	3·385
40	1·303	1·684	2·021	2·423	2·704	3·307
60	1·296	1·671	2·000	2·390	2·660	3·232
120	1·289	1·658	1·980	2·358	2·617	3·160
∞	1·282	1·645	1·960	2·326	2·576	3·090



Control Chart Coefficients

Table 1

n	2	3	4	5	6	7	8	9
a_n	0.8862	0.5908	0.4857	0.4299	0.3946	0.3698	0.3512	0.3367

Table 2

Sample Size n	2	3	4	5	6	7	8	9	10	11	12
$A_{0.025}$	1.229	0.608	0.476	0.377	0.316	0.274	0.244	0.202	0.220	0.186	0.174
$A_{0.001}$	1.937	1.054	0.750	0.594	0.498	0.432	0.384	0.347	0.317	0.294	0.274

Table 3

	For use with σ				*For use with* $\overline{w}$			
n	$D_{0.001}$	$F_{0.025}$	$F_{0.975}$	$D_{0.999}$	$D'_{0.001}$	$F'_{0.025}$	$F'_{0.975}$	$D'_{0.999}$
2	0.00	0.04	3.17	4.65	0.00	0.04	2.81	4.12
3	0.06	0.30	3.68	5.06	0.04	0.18	2.17	2.99
4	0.20	0.30	3.98	5.31	0.10	0.29	1.93	2.58
5	0.37	0.85	4.20	5.48	0.16	0.37	1.81	2.36
6	0.54	1.06	4.36	5.62	0.20	0.42	1.72	2.22
7	0.69	1.25	4.49	5.73	0.26	0.46	1.66	2.12
8	0.83	1.41	4.61	5.82	0.29	0.50	1.62	2.04
9	0.96	1.55	4.70	5.90	0.32	0.52	1.58	1.99
10	1.08	1.67	4.79	5.97	0.35	0.54	1.56	1.94
11	1.20	1.78	4.86	6.04	0.38	0.56	1.53	1.90
12	1.30	1.88	4.92	6.09	0.40	0.58	1.51	1.87