CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2011

Module Title: Technological Mathematics 2

Module Code: **MATH 6046**

School: Electrical & Electronic Engineering

Programme Title: Bachelor of Engineering in Electronics
Bachelor of Engineering (Honours) in Electronic Systems Engineering

Programme Code: **EELXE_7_Y1**
EELES_8_Y1

External Examiner(s): **Dr. P. Kirwan**
Internal Examiner(s): **Dr. P. O'Connor**

Instructions: **Answer QUESTION 1 (worth 40 marks) and TWO other questions (worth 30 marks each)**

Duration: 2 Hours

Sitting: Summer 2011

Requirements for this examination: Graph paper, Log Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

1. (a) Given z_1 = (6+ j4) and z_2 = (7 – j2) determine $\frac{z_1}{z_2}$ in Cartesian form and in polar form. Verify your answers are the same.

(5 marks)

(b) If $z_3 = [8(cos\frac{\pi}{6} + jsin\frac{\pi}{6})]^5$ and $z_4 = [12(cos\frac{\pi}{4} + jsin\frac{\pi}{4})]^3$, express $\frac{z_3}{z_4}$ in (a + jb) form.

(5 marks)

(c) The voltage (V) volts in a circuit at time (t) is given by

$$V(t) = 7(3 - 2.e^{\frac{-4t}{5}}).$$

Find an expression for $\frac{dV}{dt}$.

Hence evaluate $\frac{dV}{dt}$ at

(a) t = 0 and (b) t = 10 seconds.

(5 marks)

(d) Show that $y = e^{-3x}$ is a solution to the differential equation

$$y'' + 4y' + 3y = 0$$

(5 marks)

(e) Determine the equation of the tangent line to the curve y at the point x = 2.

$$y = \mathrm{e}^{(\frac{-2\mathrm{x}}{5})} + \frac{\mathrm{x}}{6}$$

(5 marks)

(f) Evaluate $\int_0^3 (\frac{2x^3+4x^2-7x}{x})dx$. (5 marks)

(g) Evaluate the integral $\int_0^2 \frac{3x^2}{\sqrt{1+x^3}}dx.$ (5 marks)

(h) The acceleration a ms^{-2} of an object is given by $a = \frac{t}{5} + 7$ where t is the time in seconds. Derive expressions for the velocity v ms^{-1} and the displacement x m if the initial velocity and displacement are both zero.

(5 marks)

2. (a) If $z = \frac{-4-j\frac{1}{\sqrt{3}}}{4+j\frac{1}{\sqrt{3}}}$, reduce z to its simplest Cartesian (a+jb) and polar (r ‹θ)
forms. Hence, show that

$$\sqrt{Z} = j$$

(8 marks)

(b) Express $z_1 = (\frac{2}{\sqrt{2}} - j\,\frac{\sqrt{3}}{3})$ in polar form, hence evaluate

$$z_2 = (\frac{2}{\sqrt{2}} - j\,\frac{\sqrt{3}}{3})^4$$

(5 marks)

(c) Convert Z_3 to its simplest Polar form ($r\langle\theta$)

$$Z_3 = \frac{\{2\langle 30^0\} + \{5\langle -45^0\}}{\{-8\langle -60^0\} - \{14\langle 80^0\}}$$

(5 marks)

(d) A resistance of 90Ω is connected in series with a 120mH inductor.
If the applied voltage is 230V at 50Hz, determine

(i) the inductive reactance
(ii) the magnitude of the impedance and its phase angle
(iii) the current and its phase angle relative to the applied voltage
(iv) the voltage across the resistor
(v) the voltage across the inductor
(vi) a phase diagram for the system.

(12 marks)

3. (a) Differentiate each of the following:

(i) $y = 7\sqrt{x^3} + \frac{5}{x^4} - 3\cos(2x - 1) + \ln(6x^2 - 3)$

(ii) $y = e^{-2x}.\cos(\pi x + 0.1)$

(iii) $y = \frac{(-5x+1)^{-3}}{(x-2)}$

(iv) $y = \sqrt{8x^3 - 3x^2 - x}$

(16 marks)

(b) An electrical voltage v(t) is given as

$$v(t) = -17\sin(150\pi t) - 12\cos(150\pi t)$$

where t is the time in seconds.

(i) Determine the time (t>0) for the first voltage **minimum** and calculate the voltage at this time.

(ii) Draw a rough sketch of v(t) over one cycle.

(14 marks)

4. (a) Evaluate the following integrals:

(i) $\int\{ 9x^5 - \frac{6}{x^4} + \cos(3-2x) + 4e^{-2x}\}dx$

(ii) $\int_2^3 \left\{ \frac{4x^3+x^2-3x}{2x^2} \right\} dx$

(iii) $\int_0^{\frac{\pi}{3}} 5sin^3x.cosx\ dx$

(iv) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} 3sin^2(2x)\ dx$

(v) $\int_0^{3/2} \frac{1}{\sqrt{9-4x^2}}\ dx$

(20 marks)

(b) A sinusoidal voltage, $v = 230\sin(\omega t\ \text{-}\theta)$volts is applied to a circuit.
Use integration to determine over **half** a cycle
(i) the mean value if θ=0
(ii) the mean value if θ= π/2
Use a sketch to explain the difference in values from (i) and (ii)

(10 marks)

5. (a) Differentiate $f(x) = \sqrt{x+1}$ from first principles.

(11 marks)

(b) Use Newton's method to determine the positive root of the Cubic Equation $x^3 - x^2 - 85 = 0$, correct to two significant figures.

(11 marks)

(c) Find the particular solution of the second order differential equation $\frac{d^2y}{dx^2} = -3x - 8$ given that $y(0) = 6$ and $y'(0) = -2$

(8 marks)