CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2010/2011

Module Title: MATH7005: Engineering Mathematics 202

Module Code: **MATH7005**

School: School of Mechanical & Process Engineering
School of Manufacturing, Biomedical & Facilities Engineering

Programme Title: **Bachelor of Engineering (Honours) in Mechanical Engineering**
Bachelor of Engineering (Honours) in Biomedical Engineering

Programme Code: **EMECH-8-Y2**
EBIOM-8-Y2

External Examiner(s): **Dr.P.Robinson**
Internal Examiner(s): **Mr. T. O Leary**

Instructions: **Select any four questions. The questions carry equal marks.**

Duration: 2 Hours

Sitting: Summer 2011

Requirements for this examination:

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

1. The displacement x of a mass attached to a spring and a dashpot at any instant t is found by solving the differential equation

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=f(t) \qquad x(0)=x'(0)=0$$

By using Laplace Transforms solve this differential equation in the cases where

(i) m=1, c=4, k=4, f(t)=60δ(t-1), (3 marks)

(ii) m=1, c=0, k=9and f(t)=36cos3t, (6 marks)

(iii) m=1, c=0, k=9 and f(t)=36(t-1)U(t-1)-72(t-2)U(t-2) (9 marks)

(iv) m=1, c=0, k=0 and f(t) is defined by

$$f(t)=\begin{cases} 3t & \text{if } 0\le t<2 \\ 6-3t & \text{if } t>2 \end{cases}$$ (7 marks)

2. (a) Green's Theorem states: If C is a piecewise smooth closed curve that encloses a region R and if f(x,y) and g(x,y) have continuous partial derivatives throughout R then

$$\oint_C f(x,y)dx+g(x,y)dy=\iint_R \left(\frac{\partial g}{\partial x}-\frac{\partial f}{\partial y}\right)dA$$

where the direction of C is anticlockwise.

Verify Greens Theorem where $f(x,y)=6y^2$, g(x,y)=18xy and R is the triangular region with vertices (-1,0), (1,0) and (1,4). (12 marks)

(b) The region R is the elliptical region defined by

$$\frac{x^2}{4}+\frac{y^2}{9}\le 1$$

(i) If C is the perimeter of this region evaluate the line integral

$$\oint_C 3xydx+12y^2dy$$

(ii) By evaluating an appropriate double integral find the second moment of area of this region about the y-axis. (13 marks)

3. (a) Find the eigenvalues and the corresponding eigenvectors of the matrices

$$\mathbf{A}=\begin{pmatrix} 3 & 2 & 1 \\ 4 & 4 & 4 \\ 1 & 2 & 3 \end{pmatrix} \qquad \mathbf{B}=\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 1 & 2 & 0 \end{pmatrix}$$

Deduce whether or not that the eigenvectors of the matrix **A** are linearly independent and mutually orthogonal. Does a non-singular matrix **H** exist where $\mathbf{H}^{-1}\mathbf{AH}$ is diagonal? Justify your answer. If it exists write down the matrix **H** and the diagonal matrix.

(17 marks)

(b) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{pmatrix} 5 & 1 \\ -1 & 3 \end{pmatrix}$$

Hence or otherwise find the general solution of the set of simultaneous differential Equations

$$\frac{dx}{dt} = 5x + y$$
$$\frac{dy}{dt} = -x + 3y$$

(8 marks)

4. (a) A volume V is described by V: $x^2+y^2 \leq 9 \quad 0 \leq z \leq 2$

(i) If C is the perimeter of the base evaluate the line integral

$$\oint_C 2x\,dx + 4y\,dy$$

(ii) Evaluate the surface integral below over the base and over the top surface of V

$$\iint_S 4x^2 z\,dS$$

(iii) Evaluate the triple integral below over this volume

$$\iiint_V (4x^2 z + 4y^2 z)\,dV$$

(16 marks)

(b) The displacements of three masses from their equilibrium positions are found by solving the system of differential equations

$$x_1'' = 12x_1 - 3x_2 - 12x_3$$
$$x_2'' = 12x_1 - 9x_2 - 6x_3 \qquad \text{(9 marks)}$$
$$x_3'' = 21x_1 - 3x_2 - 21x_3$$

By assuming solutions of the from $x_i = R_i \cos(\omega t - \alpha i)$ solve this set of equations.

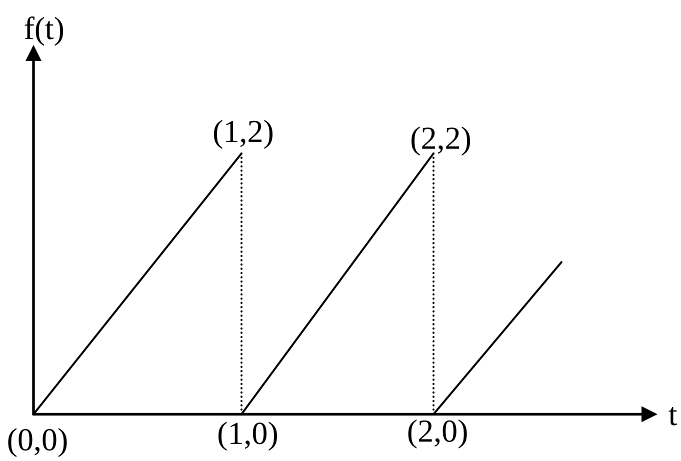
5 (a) By using partial fractions and by using long division find the first four sampled values whose z-transform is given by

$$\frac{4z^2}{(z-2)(z-1)^2} = \frac{4z^2}{(z^3-4z^2+5z-2)} \qquad \text{(10 marks)}$$

(b) Use z-Transforms to solve the difference equations

$$y_{n+2}-4y_{n+1}+4y_n=0 \qquad y_0=6 \qquad y_1=0 \qquad \text{(7 marks)}$$

(c) Find the Laplace Transform of **two** cycles of the sawtooth wave below



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(8 marks)

Z-TRANSFORMS

f(t)	F(z)
U(n)=1	$\frac{z}{z-1}$
a^N	$\frac{z}{z-a}$
n	$\frac{z}{(z-1)^2}$
n^2	$\frac{z(z+1)}{(z-1)^3}$
$a^n f(n)$	$F\left(\frac{z}{a}\right)$
nf(n)	-zF(z)
f(n+1)	zF(z)-zf(0)
f(n+2)	$z^2F(z)-z^2f(0)-zf(1)$

DERIVATIVES AND INTEGRALS

f(x) a=constant	**$f'(x)$**
x^n	nx^{n-1}
e^{ax}	$a\,e^{ax}$
sinx	cosx
cosx	-sinx

f(x) a=constant	**$\int f(x)dx$**
x^n	$\frac{x^{n+1}}{n+1}$ if $n \neq -1$
e^{ax}	$\frac{1}{a}\,a\,e^{ax}$
sinx	-cosx
cosx	sinx

Note: 2sinAcosB=sin(A+B)+sin(A-B) 2cosAcosB=cos(A+B)+cos(A-B)

2sinAsinB=cos(A-B)-cos(A+B) sin(-A)=-sinA cos(-A)=cosA

$\cos^2A=\frac{1}{2}(1+\cos2A)$ $\sin^2A=\frac{1}{2}(1-\cos2A)$

LAPLACE TRANSFORMS

For a function f(t) the Laplace Transform of f(t) is a function in s defined by

$$F(s) = \int_0^{\infty} e^{-st} f(t)\,dt \quad \text{where } s>0.$$

f(t)	F(s)
A=constant	$\frac{A}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
sinhkt	$\frac{k}{s^2-k^2}$
s	$\frac{s}{s^2-k^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at} f(t)$	F(s-a)
$f'(t)$	sF(s)-f(0)
$f''(t)$	$s^2F(s) - sf(0) - f'(o)$
$\int_0^t f(u)du$	$\frac{F(s)}{s}$
$\int_0^t f(u)g(t-u)du$	F(s)G(s)
U(t-a)	$\frac{e^{-as}}{s}$
f(t-a)U(t-a)	$e^{-as}F(s)$
$\delta(t-a)$	e^{-as}

Note: $\cosh A = \frac{e^A + e^{-A}}{2}$ $\qquad$ $\sinh A = \frac{e^A - e^{-A}}{2}$