

**CORK INSTITUTE OF TECHNOLOGY  
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ**

**Semester 2 Examinations 2010/11**

**Module Title: Calculus & Statistical Analysis**

**Module Code:** MATH 8001

**School:** Science

**Programme Title:**

Bachelor of Science (Honours) in Analytical Chemistry with Quality Assurance – Year 2

**Programme Code:** SCHQA\_8\_Y2

**External Examiner(s):** Dr. P. Kirwan

**Internal Examiner(s):** Ms. F. Wood

**Instructions:** Answer question 1 worth 40 marks.  
Answer ONE question from Section A and ONE question from Section B.  
Total marks = 100

**Duration:** 2 Hours

**Sitting:** Summer 2011

**Requirements for this examination:** Mathematics Tables

**Note to Candidates:** Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.

If in doubt please contact an Invigilator.

Q.1(a) A curve is described in parametric form by  $x = t^2$ ,  $y = \frac{1}{t^2}$ . Find  $\frac{dy}{dx}$  for the function and hence find the equation of the tangent to the curve at  $t = 3$ .

(8 marks)

(b) The modulus of rigidity  $G$  is defined as  $G = \frac{R^4\theta}{L}$  where  $R$  = radius,  $\theta$  = angle of twist and  $L$  = length. Determine the approximate percentage change in  $G$  when  $R$  increases by 1% and  $L$  increases by 2%.  $\theta$  remains constant.

(8 marks)

(c) Determine

$$\int x^3 \ln(x) dx$$

(8 marks)

(d) It is found that 28% of blood samples taken from people suffering muscle cramp have 'high' levels of potassium. Blood samples are taken from 12 randomly selected patients. Find the probability that

- three samples
- at least two samples

have 'high' potassium levels.

(8 marks)

(e) A survey of a college computer system helpdesk was carried out. A random sample of 180 requests to the helpdesk showed that 54 of them were taking 15 minutes or more to process. Find a 95% confidence interval for the proportion of all requests taking at least 15 minutes to process. Interpret your result.

(8 marks)

## Section A

Q.2(a) Show that the point  $(2, -1)$  lies on the curve  $3x^2 - 4y^3 = 24 - x^3y^2$

Find  $\frac{dy}{dx}$  for the function and find the slope of the tangent to the curve at this point.

Find where the function crosses the  $x$ -axis.

(12 marks)

(b) If

$$V = \ln(x^2 + y^2)$$

find partial derivatives  $\frac{\partial V}{\partial x}$  and  $\frac{\partial V}{\partial y}$ . Express  $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y}$  in its simplest form.

Find  $\frac{\partial^2 V}{\partial x \partial y}$ .

(8 marks)

(c) The blood pressure  $P$ (mm Hg) of a woman varies with age  $t$  (years) and level of physical exercise  $E$  according to the equation

$$P = 217 - 0.85t - \frac{7E}{t}$$

- (i) What is the blood pressure of a 45 year old woman whose exercise level is 100?
- (ii) Determine her rate of change of blood pressure with respect to age at this point.
- (iii) Determine her rate of change of blood pressure with respect to exercise at this point.

(10 marks)

Q.3(a) Evaluate the following integral:

$$\int_3^4 \frac{2x}{\sqrt{x^2-5}} dx$$

(8 marks)

(b) Find the general solution to the following differential equation, giving the solution in exponential form.

$$\frac{dy}{dx} = -2x(y+1)$$

(6 marks)

(c) The rate of a second order chemical reaction is given by

$$\frac{d[A]}{dt} = -k[A]^2$$

Solve the equation to give the integrated rate law given that  $[A] = [A_0]$  when  $t = 0$  and  $[A] = [A]$  when  $t = t$ .

(7 marks)

(d) The displacement ( $x$ ) of a vibrating particle is described by the differential equation

$$\frac{d^2x}{dt^2} + 9x = 0$$

where  $t =$  time. Solve the equation given that  $x = 15$  and  $\frac{dx}{dt} = 0$  when  $t = 0$ .

(9 marks)

## Section B

Q.4(a) The development of Free Fatty Acids (FFA) in herring stored at  $6^{\circ}\text{C}$  was studied during a 20-day period. The results are tabulated as follows:

No. of days storage ( $x$ )	1	7	12	15	20
FFA (%) ( $y$ )	3.2	12.8	16.4	28.0	37.1

- (i) Plot the data on a scatter diagram. (5 marks)
- (ii) Calculate the equation of the regression line of % FFA on storage and fit this line to your graph in (i). (12 marks)
- (iii) Calculate a coefficient of correlation for the data. Interpret your result. (5 marks)
- (b) A company providing maintenance for laboratory equipment receives on average 18 callouts per 5-day week. Calculate the probability that on any given day the centre will receive
- (i) less than three
- (ii) more than three callouts. (Assume a Poisson Distribution) (8 marks)

- Q.5(a) The duration of telephone calls to a company has been found to be approximately normally distributed with a mean duration of 4.8 minutes and a standard deviation of 1.2 minutes. Calculate the proportion of calls that have a duration of
- more than six minutes
  - less than two minutes
  - between 2 and 5 minutes.

(11 marks)

- (b) The following table shows the protein content in a sample of seven brands of baking flour:

Brand	A	B	C	D	E	F	G
g /100g	10.0	10.4	9.8	11.2	9.3	12.1	10.6

- Calculate the mean and standard deviation of the data.
- Calculate a 95% confidence interval for the population mean and interpret your result.

(10 marks)

- (c) A chemical plant is producing cartons of a substance and the mean carton weight is supposed to be 1kg. The production manager suspects that the filling machine is underweighing. A random sample of 30 cartons shows a mean of 0.989kg and a standard deviation of 0.028kg. Test the manager's suspicion at the 5% significance level and at the 1% significance level and interpret your results.

(9 marks)

## Statistical Formulae

### 1. Descriptive Statistics

$$\text{Mean: } \bar{x} = \frac{\sum x_i}{n} \qquad \text{Standard Deviation: } s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

### 2. Regression and Correlation

$$y' = a + bx$$

$$\begin{aligned} \sum y &= na + b\sum x \\ \sum xy &= a\sum x + b\sum x^2 \end{aligned} \qquad \bar{y} = a + b\bar{x}$$

$$a = \frac{\sum y - b\sum x}{n} \qquad b = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - (\sum x)^2}$$

Coefficient of Correlation:

$$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \cdot \sqrt{n\sum y^2 - (\sum y)^2}}$$

### 3. Probability Distributions

(i) Binomial Distribution

$$P(r,n) = {}^n C_r p^r q^{n-r}$$

(ii) Poisson Distribution

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!}$$

(iii) Normal Distribution

Standard Units

$$z = \frac{x - \bar{x}}{s} = \frac{x - \mu}{\sigma}$$

### 4. Sampling

$$\text{Standard Error of the mean} = \frac{s}{\sqrt{n}}$$

$$\text{Standard Error of the proportion} = \sqrt{\frac{pq}{n}}$$

$$Z\text{-score} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

## Further Relevant Equations

Solutions to Second Order D.E.

$$y = Ae^{m_1x} + Be^{m_2x} \quad \text{Roots real and distinct}$$

$$y = e^{m_1x}(A + Bx) \quad \text{Roots real and equal}$$

$$y = e^{\alpha x}(C \cos(\beta x) + D \sin(\beta x)) \quad \text{Complex roots}$$