CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 2 Examinations 2010/2011

Module Title: Statistical Quality Control for Chemists.

Module Code: **STAT 8003**

School: Science

Programme Title:
Bachelor of Science (Honours) in Analytical Chemistry with Quality Assurance - Award

Programme Code: **SACQA_8_Y4**

External Examiner(s): Mr. J. Reilly
Internal Examiner(s): Mr. D. O'Hare

Instructions: Answer **any three questions.** All questions carry equal marks.

Duration: 2 Hours

Sitting: Summer 2011

Requirements for this examination: **Statistical Tables by Murdoch and Barnes.**

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

Q1. (a) Consider the following acceptance sampling plan.

Take a random sample of 100 items from the batch. If the sample contains 2 or fewer defectives, accept the batch; otherwise reject it.

Determine P(A), the probability of acceptance, for each of the following batch proportion defective levels, p: 0.01, 0.02, 0.03, 0.04, 0.05. Plot the values of P(A) versus p.

(6 marks)

(b) Control charts for $\overline{\mathrm{X}}$ and R are maintained for an important quality characteristic. The sample size is n=7. After 35 samples, the following information is available:

$\sum_1^{35} \bar{x} = 7805$ and $\sum_1^{35} R = 1200.$

(i) Establish the limits for $\overline{\mathrm{X}}$- and R- charts.

(ii) Assuming that both charts exhibit a state of statistical control, estimate the process mean and standard deviation.

(iii) If the quality characteristic is normally distributed, and if the specifications are 220 ± 30, find the fraction nonconforming for the process.

(iv) Assuming the variance to remain constant, state where the process mean should be located to minimise the fraction nonconforming, and find the fraction nonconforming in that situation.

(12 marks)

(c) A control chart for the proportion defective indicates that the current process average is 0.03. A sample of size 200 is in use.

(i) Establish the three-sigma limits for the chart.

(ii) What is the probability that a shift in the process average to 0.07 will be detected on the first sample subsequent to the shift? What is the probability that this shift will be detected by no later than the fourth sample following the shift?

(7 marks)

Q2. (a) Batches of items are monitored for the concentration of a certain impurity. The mean concentration varies from batch to batch, but the standard deviation for a batch can be assumed to be constant at 6ppm.

Suppose an acceptance sampling scheme for these batches reads: '*select 20 items at random and determine the mean impurity concentration for the selected items. Accept the batch if the sample mean is less than 18 ppm, otherwise reject it*'.

(i) Describe the distribution of means of samples of size 20 taken from a batch where the mean concentration of impurity is 15 ppm.

(ii) Find the probability that such a batch will be accepted by the plan. (5 marks)

(b) The following results were obtained by an analyst using a new method for the determination of nickel in a standard reference alloy containing 6% nickel:

6.01, 6.25, 5.88, 6.31, 6.27, 6.33

(i) Determine a 95% confidence interval for the **variance** of measurements made by the analyst.

(ii) Does the mean of the results obtained by the analyst differ significantly from the standard figure of 6%? Support your answer by carrying out a suitable test of hypothesis.

(8 marks)

(c) Two different etching solutions are compared with respect to their respective etch rates. The observed etch rates for the two solutions are as follows:

Solution 1	0.251	0.239	0.245	0.259	0.269	0.262	0.254
Solution 2	0.257	0.269	0.272	0.265	0.267	0.262	0.272

Do the data support the claim that the mean etch rate is the same for both solutions? Justify your answer by conducting the appropriate tests of hypothesis.

(12 marks)

Q3. (a) An experiment was designed to study the effects of melt flow index (rate) and filler content (% of weight) on the strength of resin. The following data were obtained.

Melt flow index	Filler content	Replicate 1	Replicate 2
3	32	1.95	2.14
3	37	2.36	2.28
3	42	2.61	2.56
12	32	2.32	2.31
12	37	2.33	2.28
12	42	2.60	2.53

(i) Produce the analysis of variance table for this set of data, and state your conclusions.
Note that the error sum of squares is 0.026250 and the interaction sum of squares is 0.055117.
(ii) Draw an interaction plot and comment on how it relates to your conclusions in part (i).

(12 marks)

(b) The yields (in percent) of a certain chemical process where different levels of temperature, reaction time, and concentration are involved are under investigation. The mean values of the response variable shown for each combination of factor levels are based on three replicates in each case.

Temperature (°F)	Time (minutes)	Concentration (%)	$\bar{y}$
250	50	3	71.8
200	50	5	81.5
250	60	5	71.2
250	50	5	78.8
200	50	3	77.2
200	60	3	74.2
200	60	5	84.8
250	60	3	67.5

(i) Label the levels of each of the factors as '-' or '+' and rewrite the table in standard order.
(ii) Estimate the temperature effect and the temperature-time interaction effect.
(iii) The pooled estimate of error variance is $s_e^2 = 24$. Test the significance of the estimated effects.

(13 marks)

Q4. The following data were obtained in the course of an experiment designed to study the relationship between temperature and turbidity.

Temperature (*x*)	22.9	24.0	22.9	23.0	20.5	26.2	25.8
Turbidity (*y*)	125	118	103	105	76	122	135

$\sum x = 165.3 \quad \sum y = 784.0 \quad \sum xy = 18706 \quad \sum x^2 = 3926.1 \quad \sum y^2 = 90068$

(i) Draw a scatter-plot, draw a straight line by eye, and use it to predict the turbidity corresponding to a temperature of 25^0C. (5 marks)

(ii) Find the equation of the regression line of turbidity on temperature. (8 marks)

(iii) Find 95% confidence limits for the slope of the true regression line. (4 marks)

(iv) Fill in the missing entries in the ANOVA table below, and say what conclusion may be drawn from it. Also use it to find the value of the correlation coefficient. (8 marks)

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                         Analysis of Variance

Source          DF      SS       MS       F      P
Regression       *       *        *       *      *
Residual Error   *    636.6       *
Total            *   2260.0
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Statistical formulae.

Sampling

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$$

$$t_{n-1} = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$$

$$\bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}};$$

$$\bar{X} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}.$$

$$z = \frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^{\ 2}}{n_1}+\frac{\sigma_2^{\ 2}}{n_2}}}$$

$$t_{n_1+n_2-2} = \frac{(\overline{x_1}-\overline{x_2})-(\mu_1-\mu_2)}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$s_p^{\ 2} = \frac{(n_1-1)s_1^{\ 2}+(n_2-1)s_2^{\ 2}}{n_1+n_2-2}$$

$$\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{s_1^{\ 2}}{n_1}+\frac{s_2^{\ 2}}{n_2}}} \sim t_\nu$$

$$\nu = \frac{\left(\frac{s_1^{\ 2}}{n_1}+\frac{s_2^{\ 2}}{n_2}\right)^2}{\frac{(s_1^{\ 2}/n_1)^2}{n_1+1}+\frac{(s_2^{\ 2}/n_2)^2}{n_2+1}}-2$$

$$\chi_{n-1}^{\ 2} = \frac{(n-1)s^2}{\sigma^2}$$

$$F_{n_2-1}^{n_1-1} = \frac{s_1^{\ 2}}{s_2^{\ 2}}$$

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}\right)$$

$$SE(p) = \sqrt{\frac{p(1-p)}{n}}$$

$$SE(c) = \sqrt{c}$$

ANOVA

1. **One-way model**: $y_{ij} = \mu + \tau_i + \varepsilon_{ij}$, $i = 1,2,\ldots,a$, $j = 1,2,\ldots.,n.$

Total SS: $\sum\sum y_{ij}{}^2 - \frac{(y_{..})^2}{an}$

Factor SS: $\sum_{i=1}^{a} \frac{y_{i.}^2}{n} - \frac{(y_{..})^2}{an}$

2. **AXB factorial model**:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta_{ij}) + \varepsilon_{ijk},\ i = 1,2,\ldots,a;\ j = 1,2,\ldots,b;\ k = 1,2,\ldots,n.$$

Total SS: $\sum\sum\sum\ y^2{}_{ijk} - \frac{y_{...}^2}{abn}$

SSA: $\sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$

SSB: $\sum_{j=1}^{b} \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$

3. 2^k design, n replicates.

Effect estimate given by $\frac{(Contrast)}{n.2^{k-1}}$

Effect SS given by $\frac{(Contrast)^2}{n.2^k}$

Standard error of an effect estimate is $\sqrt{\frac{\sigma_e^2}{n.2^{k-2}}}$, where σ_e^2 is the error variance.

Regression

$$S_{xx} = \sum x^2 - \frac{(\sum x)^2}{n} \qquad S_{xy} = \sum xy - \frac{\sum x \sum y}{n} \qquad S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} \qquad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \qquad s_e = \hat{\sigma} = \sqrt{\frac{S_{yy} - \hat{\beta}_1{}^2 S_{xx}}{n-2}}$$

$$S.E.(\hat{\beta}_1) = \frac{s_e}{\sqrt{S_{xx}}} \qquad S.E.(\hat{\beta}_0) = s_e \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}} \qquad SSR = \frac{(S_{xy})^2}{S_{xx}}$$

$$S.E.(\hat{\beta}_0 + \hat{\beta}_1 x_0) = s_e \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}} \qquad S.E.(individual\,\hat{\beta}_0 + \hat{\beta}_1 x_0) = s_e \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$$

Inverse Prediction Formula

$$\left(\frac{\bar{Y} - \hat{\beta}_0}{\hat{\beta}_1}\right) \pm t \frac{s_e}{\hat{\beta}_1} \sqrt{\frac{1}{m} + \frac{1}{n} + \frac{(\bar{Y} - \bar{y})^2}{\hat{\beta}_1{}^2 S_{xx}}}$$

$$t_{n-2} = \frac{\hat{\beta}_1 - \beta_{1,0}}{SE(\hat{\beta}_1)} \qquad t_{n-2} = \frac{\hat{\beta}_0 - \beta_{0,0}}{SE(\hat{\beta}_0)}$$