CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/2018

Module Title: Calculus & Statistics

Module Code: **MATH 6002**

School: School of Science and Informatics

Programme Title:

B.Sc. (Hons.) in Pharmaceutical Biotechnology – Year 1
Common Entry (Hons) Biological Sciences – Year 1

Programme Code:

SPHB_8_Y1 **SCEBS_8_Y1**

External Examiner: **Dr. J. Cruickshank**
Internal Examiner: **Ms. M. Brennan, Ms. H. Lordan**

Instructions: **Answer All Questions.**
Show all calculations in full.

Duration: **2 HOURS**
Sitting: Autumn 2018
Requirements for this examination: Graph paper, Mathematical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

1. (a) Differentiate $f(x) = x^2 - 3x + 1$ from first principles. [6 marks]

(b) Differentiate each of the following functions.

(i) $y = \dfrac{e^{-8t} - 2t^4}{6t^{-\frac{1}{2}} + \sin t}$ [6 marks]

(ii) $y = (\cos x + \ln x - x^{-5} + 9)^3$ [6 marks]

(c) The number of cells in a culture at time t hours is modelled by the equation

$$N(t) = 180e^{0.5t},\ t \geq 0$$

(i) Find the number of cells in the culture after 4 hours.

(ii) Determine the rate of change in the number of cells after 12 hours.

Don't forget your units. [5 marks]

(d) Consider the function given by $y = \dfrac{5}{3}x^3 - \dfrac{5}{2}x^2 + 8$.

(i) Find the turning points of the function.

(ii) Use the Second Derivative Rule to classify the turning points as either maximum or minimum points.

(iii) Determine the point of inflection.

(iv) Draw a rough sketch of the graph of y. [10 marks]

2. (a) Evaluate the following integrals.

(i) $\int \left(x^5 + \frac{1}{x^3} - \sqrt{x} + \frac{2}{x}\right) dx$ [6 marks]

(ii) $\int_2^4 (3t^2 + 2t)\sqrt{t^3 + t^2}\, dt$ [6 marks]

(iii) $\int (\cos x)e^{\sin x}\, dx$ [4 marks]

(b) Let $y = 12 - x^2$ and $y = x^2 + 4$ be two functions.

(i) Find the points of intersection of the two functions. [5 marks]

(ii) Draw the two curves from $x = -3$ to $x = 3$ and shade in the region bounded by them. [4 marks]

(iii) Find the area of the shaded region. [8 marks]

3. A sample of 80 biscuit and cereal bars were analysed for carbohydrate content. The findings were tabulated as follows:

Carbohydrate Content (g *per* $100g$)	No. of Bars
60.0-63.9	5
64.0-65.9	11
66.0-67.9	19
68.0-69.9	24
70.0-73.9	15
74.0-77.9	6

(a) Calculate the mean carbohydrate content ($\overline{x}$) and the standard deviation (s) from the mean. *Please refer to formula sheet for* $\overline{x}$ *and* s. [16 marks]

(b) Represent the data on a histogram. [10 marks]

Use the histogram to estimate

(i) the mode and the median, and [4 marks]

(ii) the percentage of samples lying in the range $(\overline{x} - s)$ to $(\overline{x} + s)$. [4 marks]

Standard Derivatives

y=f(x)	$\frac{dy}{dx}=f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$u \cdot v$	$u\frac{dv}{dx}+v\frac{du}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
$y(u(x))$	$\frac{dy}{du}\cdot\frac{du}{dx}$

Standard Integrals

$y=f(x)$	$\int f(x)dx$
x^n $(n\neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln\lvert x\rvert$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$

Statistical Formulae

Mean: $\bar{x}=\frac{\sum fx}{\sum f}$

Standard deviation: $s=\sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f-1}}$