CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Repeat Semester 1 & Semester 2 Examinations 2017/18

Module Title: Discrete Mathematics with Linear Algebra

Module Code: **MATH 6004**

School: Science and Informatics

Programme Title:
Bachelor of Science (Honours) in Software Development - Year 1/Year 2
Bachelor of Science (Honours) in Computer Systems - Year 1/Year 2
Bachelor of Science in Software Development - Year 1/Year 2
Bachelor of Science (Honours) in Web Development - Year 1
Bachelor of Science in Information Technology - Year 1
Bachelor of Science in (Honours) in IT Management - Year 1
Higher Certificate in Software Development - Year1

Programme Code:
KSDEV_8_Y1/2, KDNET_8_Y1/Y2, KCOMP_7_Y1/2, KWEBD_8_Y1, KITSP_7, KITMN_8_Y1, KCOM_6_Y1

External Examiner(s): **Dr. Ann O'Shea**
Internal Examiner(s): **Dr. Michael Brennan**
Dr. Robert Heffernan
Dr. Justin McGuinness
Dr. Marie Nicholson
Mr. Adrian O'Connor

Instructions:
Answer all questions.

Duration: **2 HOURS**

Sitting: Autumn 2018

Requirements for this examination:

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator

Q1. (a) State the rule of inference known as *Resolution* using propositional variables p, q and r.

(4 marks)

(b) Using *laws of logic* simplify the expression as far as possible. Indicate explicitly the laws that are used in each step.

$$(\neg p \vee q) \rightarrow \neg p$$

(12 marks)

(c) Write the following argument in symbolic logic. State whether it represents a valid argument or a fallacy. Quote the appropriate rule of inference or fallacy.

Mary's laptop is either on the table or in the computer bag. Mary's computer is not on the table. Therefore Mary's laptop is in the computer bag.

(8 marks)

(d) The argument $(q \wedge (p \rightarrow q)) \rightarrow p$ is a *fallacy*.

(i) Identify this *fallacy*;

(ii) Construct a counterexample illustrating the *fallacy*.

$$\begin{array}{c} q \\ p \rightarrow q \\ \hline \therefore p \end{array}$$

(6 marks)

(e) Use the laws of sets to simplify the expression $A \cap \overline{(\overline{A} \cup B)}$. Indicate explicitly the laws that are used in each step.

(8 marks)

(f) Let $A = \begin{bmatrix} 4 & -2 \\ 6 & -1 \end{bmatrix}, B = \begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$.

Determine $AB + 3C$.

(8 marks)

Q1 continued on next page

Q1.(g) The following augmented matrix represents a linear 4×4 system that has been row reduced to echelon form. Identify the variable that is free and hence determine the solution set S.

$$\left[\begin{array}{cccc|c} 1 & 0 & 5 & 0 & -6 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

(6 marks)

Q1.(h) Let $A = \{2, 4, 6, 8\}$ and the relation R on A is given by the matrix

$$\begin{bmatrix} T & F & T & T \\ F & T & F & T \\ T & F & F & F \\ F & T & T & T \end{bmatrix}$$

(i) Describe the relation R as a subset of $A \times A$.

(ii) Is R a *reflexive* relation? Explain.

(iii) Is R a *transitive* relation? Explain.

(8 marks)

Q2. (a) Use truth tables and the biconditional connective $\leftrightarrow$ to prove the *Implication Law*

$$p \to q \equiv \neg p \vee q.$$

(8 marks)

(b) Determine the solution set S for the 2×2 linear system using *only* the *Inverse Matrix Method.*

$$\begin{array}{rcrcr} 4x & + & 3y & = & -1 \\ x & + & 2y & = & 8 \end{array}$$

(8 marks)

(c) State the logic laws known as the *Idempotent laws.*

(4 marks)

Q3. (a) Write down the standard matrix A of the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ that represents the linear transformation that $T(1,0) = (2,1)$ and $T(0,1) = (-1,3)$. Hence determine $T(2,4)$

(6 marks)

(b) Let $A' = 101\,110, B' = 011\,011$ represent the bit string representation of the sets A and B respectively. Determine

(i) $(A \cap B)'$ (ii) $\overline{A}'$

(4 marks)

(c) Prove that the following argument is valid. Each step of the proof should be accompanied by the relevant rules of inference and laws of logic.

$$\begin{array}{c} p \\ \neg w \vee l \\ q \to w \wedge k \\ p \to q \\ \hline \therefore\ l \end{array}$$

(10 marks)