Cork Institute of Technology

Institiúid Teicneolaíochta Chorcaí

Autumn Examinations 2017/2018

Module Title: Engineering Maths 101

Module Code:	MATH6005
School:	Building and Civil Engineering Mechanical and Process Engineering
Programme Title:	Bachelor of Engineering (Hons) in Mechanical Engineering – Year 1 Bachelor of Engineering (Hons) in Biomedical Engineering – Year 1 Bachelor of Engineering (Hons) in Chemical and Process Engineering – Year 1 Bachelor of Engineering (Hons) Common Entry Engineering – Year 1 Bachelor of Engineering (Hons) in Structural Engineering – Year 1.
Programme Code:	EMECH_8_Y1 EBIOM_8_Y1 ECPEN_8_Y1 EOMNI_8_Y1 CSTRU_8_Y1
External Examiners(s):	Dr. A. O'Shea
Internal Examiners(s):	Dr. C. Carroll Dr. V. Morari Dr. M. Nicholson
Instructions:	Answer **all** questions. Question 1 carries 34 marks. Questions 2 and 3 carry 33 marks each. Do not write, draw or underline in RED. Show all calculations and workings in full.
Duration:	2 Hours
Sitting:	Autumn 2018
Requirements for this examination:	Mathematical Tables

Note to Candidates:

Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

Question 1.

(a) Let

$$M = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix} \text{ and } N = \begin{bmatrix} 1 & 1 & 7 \\ 8 & 2 & 2 \\ -1 & -1 & 5 \end{bmatrix}.$$

Find scalars a and b such that $aM^2 + bM^T = N$. (8 marks)

(b) Use the inverse matrix method to solve the system of equations

$$\begin{aligned} x + y + 3z &= 13 \\ x + 2y + 4z &= 16 \\ 3x + 5y + 10z &= 41 \end{aligned}$$

(14 marks)

(c) (i) Use Gaussian Elimination to find all solutions of the system of equations

$$\begin{aligned} x + y - 4z &= 3 \\ 2x - 6y \phantom{{}-2z} &= -10 \\ -x + 5y - 2z &= 9 \end{aligned}$$

(8 marks)

(ii) Find one particular solution of the system. (4 marks)

Question 2.

(a) Let $z_1 = -1 - i$ and $z_2 = 2\sqrt{3} - 2i$.

(i) Write $3z_1 + 2z_2$ as $a + bi$ where $a, b \in \mathbb{R}$. (3 marks)

(ii) Write $\frac{1}{z_2}$ as $a + bi$ where $a, b \in \mathbb{R}$. (2 marks)

(iii) Convert z_1 and z_2 to polar form. (4 marks)

(iv) Write $z_1 \cdot z_2$ in polar form. (2 marks)

(v) Write $(z_1)^{200}$ as 2^n where $n \in \mathbb{N}$. (4 marks)

(b) (i) Verify that $1 + i$ is a root of the equation $4z^3 - 7z^2 + 6z + 2 = 0$. (3 marks)

(ii) Form the quadratic equation which has $-2 + i$ as a root. (3 marks)

(c) Solve the equation $z^4 + 16 = 0$. Represent all roots on the Argand diagram. (12 marks)

Question 3.

(a) Define the vector product and the scalar (dot) product between two vectors. Give one example of an application in each case.

(6 marks)

(b) Let $\overline{a} = 2\overline{i} - \overline{j} + \overline{k}$ and $\overline{b} = \overline{i} + 3\overline{k}$ and $\overline{c} = -2\overline{i} + 2\overline{j} + \overline{k}$.

(i) Show that

$$\overline{a} \times (\overline{b} \times \overline{c}) = (\overline{a} \cdot \overline{c})\overline{b} - (\overline{a} \cdot \overline{b})\overline{c}$$

(8 marks)

(ii) Find the volume of the parallelepiped with edges represented by the vectors $\overline{a}, \overline{b}$ and $\overline{c}$. (3 marks)

(c) A force $\overrightarrow{F}$ of magnitude 100 N acts at the point $(1, 0, -5)$ in the direction of the line joining $(2, 1, 1)$ to $(4, 7, -2)$. Find

(i) the moment of force about the origin; and

(ii) the directional cosines of the force $\overrightarrow{F}$.

(8 marks)

(d) Let $\overline{a} = 2\overline{i} + \lambda\overline{j} + 3\overline{k}$ and $\overline{b} = \lambda\overline{i} - \lambda\overline{j} + \overline{k}$.

(i) For what values of λ are the vectors $\overline{a}$ and $\overline{b}$ perpendicular?

(ii) Find the angle between $\overline{a}$ and $\overline{b}$ if $\lambda = 4$.

(8 marks)