

CORK INSTITUTE OF TECHNOLOGY  
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

**Semester 2 Examinations 2017/2018**

<b>Module Title:                      Engineering Maths 102</b>
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**Module Code:** MATH6006

**School:** Building & Civil Engineering  
Mechanical & Process Engineering

**Programme Title:** Bachelor of Engineering (Hons) in Mechanical Engineering - Year 1  
Bachelor of Engineering (Hons) in Biomedical Engineering - Year 1  
Bachelor of Engineering (Hons) in Chemical & Process Engineering - Year 1  
Bachelor of Engineering (Hons) Common Entry - Year 1  
Bachelor of Engineering (Hons) in Structural Engineering - Year 1

**Programme Code:** EMECH\_8\_Y1  
EBIOM\_8\_Y1  
ECPEN\_8\_Y1  
EOMNI\_8\_Y1  
CSTRU\_8\_Y1

**External Examiners(s):** Dr A. O'Shea

**Internal Examiners(s):** Dr C. Carroll  
Dr V. Morari

**Instructions:** Answer ALL Questions.

**Duration:** 2 HOURS

**Sitting:** Autumn 2018

**Requirements for this examination:** Mathematics Tables

**Note to Candidates:**

Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

1. (a) A curve is described by the equations

$$x(t) = 4 \sin(2t) - 2, \quad y(t) = \cos^2(2t).$$

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ .

(10 marks)

- (b) Differentiate the function

$$y = (x^2 + e^{x^2})^{x^2 + e^{x^2}}$$

(9 marks)

- (c) Find the derivative of the function

$$y = 2 \cos^{-1} \left( x^2 + \frac{1}{3} \right)$$

(6 marks)

2. (a) Find the equation of the tangent to the curve

$$x^2y + 3xe^{y+1} + 2y^2 = 4x$$

at the point  $(-2, -1)$ .

(6 marks)

- (b) Let  $V = x^3e^{x-y} - 2xy^2 \sin y$ . Find

(i)

$$\frac{\partial^2 V}{\partial x^2}$$

(4 marks)

(ii)

$$\frac{\partial^2 V}{\partial y^2}$$

(4 marks)

(iii)

$$\frac{\partial^2 V}{\partial y \partial x}$$

(2 marks)

(iv)

$$\frac{\partial^2 V}{\partial x \partial y}$$

(2 marks)

- (c) Consider the function  $y = x^2 - 8x + 6 \ln x$ .

(i) Find all turning points of the function.

(4 marks)

(ii) Use differentiation to distinguish between them.

(3 marks)

3. Find each of the following integrals:

(a)

$$\int_{-3}^1 \frac{3}{\sqrt{7-6x-x^2}} dx$$

(5 marks)

(b)

$$\int \frac{3x^2 + 5x + 1}{(x+3)(x^2+4)} dx$$

(9 marks)

(c)

$$\int 2xe^{4x} dx$$

(4 marks)

(d)

$$\int_0^\pi (t^2 - 4) \sin(t^3 - 12t) dt$$

(7 marks)

4. (a) Let

$$y = 3 \sin(2x).$$

Find the volume of the solid generated by rotating the area under the curve  $y$  around the  $x$ -axis between the limits  $x = \frac{\pi}{4}$  and  $x = \frac{3\pi}{4}$ . (7 marks)

(b) Find the root mean square of the function

$$f(t) = 4t + \frac{4}{\sqrt{t}}$$

on the interval  $[1, 3]$ .

(8 marks)

(c) The equation of a curve is given by

$$y = \frac{3}{4}x^2 - \frac{1}{6} \ln 2x + \frac{5}{12}.$$

(i) Show that

$$1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{3}{2}x + \frac{1}{6x}\right)^2.$$

(4 marks)

(ii) Hence find the length of the curve on the interval  $[\frac{1}{2}, 3]$ .

(6 marks)