CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2018

Module Title: Technological Mathematics 2

Module Code: **MATH 6015**

School: School of Engineering: Building and Civil Engineering.
School of Mechanical, Electrical and Process Engineering

Programme Title: Bachelor of Engineering (Hons) in Sustainable Energy Eng. – Year 1
Bachelor of Engineering in Biomedical Eng. Year 1
Bachelor of Engineering in Civil Eng – Year 1
Bachelor of Engineering in Mechanical Eng. Year 1
Bachelor of Engineering in Environmental Eng – Year 1

Programme Code: ESENT_8_Y1
EBIME_7_Y1
CCIVL_7_Y1
EMECH_7_Y1
CENVI _7_Y1

External Examiner: **Dr. James Cruickshank.**
Internal Examiner: **Dr. C. Carroll, Ms. H. Lordan.**

Instructions: **Answer ALL Four questions.**
Clearly show all workings and calculations for full marks.

Duration: **2 HOURS**

Sitting: **Autumn 2018**

Requirements for this examination: Graph paper, Formulae & Tables Book

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.

Q1 (a) Differentiate from first principles $f(x) = x^2 - 3x + 5$

No marks will be awarded if any other method is used. [7 marks]

(b) Differentiate each of the following by rule:

(i) $y = (3x^2 - 2x^{-5} + 10)^4$ [2 marks]

(ii) $y = e^{3x}(7x - 5)$ [4 marks]

(iii) If $y = \dfrac{\sin x}{\cos x - 1}$ evaluate $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{3}$ [6 marks]

(c) Find the equation of the tangent to the curve $y = \frac{1}{\sqrt{12-3x}}$ at the point $\left(1, \frac{1}{3}\right)$.

[6 marks]

Q2 (a) (i) Find the turning points on the curve $y = 3 + 3x^2 - x^3$. [4 marks]

(ii) Use differentiation to classify them. [3 marks]

(iii) Use this information to sketch the curve. [2 marks]

(b) The total surface area of a cylinder of radius r and perpendicular height h is $54\pi\ cm^2$. Show that the volume of the cylinder may be written: $V = 27\pi r - \pi r^3$. Verify that the volume is at a maximum when the height is equal to the diameter.

[9 marks]

(c) The distance, s meters, travelled by a car t seconds after the brakes are applied is given by $s = 22t - 2.6t^2$. Calculate:

(i) the velocity of the car at the instant the brakes are applied. [3 marks]

(ii) the stopping distance of the car (i.e. the distance travelled by the car from the moment the brakes are applied to when the car stops). [4 marks]

Q3 (a) Integrate each of the following:

(i) $$\int \left(\frac{5}{x^2} - e^{5x} + x + 10\right) dx$$ [2 marks]

(ii) $$\int_0^{\frac{\pi}{2}} \sin x \, e^{\cos x} \, dx$$ [6 marks]

(iii) $$\int \frac{x^2 - 3x + 3}{(x-1)(x-2)(x-3)} dx$$ [7 marks]

(iv) $$\int_3^4 \frac{20}{x^2 + 4} dx$$ [2 marks]

(b) Use integration to find the area enclosed between the curve $y = 2x(x^2 - 1)^3$, the x-axis and the lines $x = -0.5$ and $x = 0.5$. [8 marks]

Q4 (a) The bending moment $M(x)$ of a beam satisfies the differential equation

$$\frac{dM}{dx} = -3(19 - 13x).$$

Find an expression for $M(x)$ if $M = 0$ when $x = 0$. [8 marks]

(b) Find the root mean square of the function $f(t) = 4\sin t$ on the interval $[0, \pi]$.

[9 marks]

(c) Calculate the volume generated when the area under the curve $4x^2 + 4y^2 = 100$ is rotated about the x-axis between the values $x = -5$ and $x = 5$. [8 marks]

Useful Formulae

$$V = \pi \int y^2\, dx$$

$$W = \int F\, dx$$

$$\bar{y} = \frac{1}{b-a}\int_a^b y\, dx$$

$$y_{RMS} = \sqrt{\frac{1}{b-a}\int_a^b y^2\, dx}$$