CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/2018

Module Title: Technological Mathematics 2 & Maple

Module Code: **MATH 6019**

School: Science and Informatics

Programme Title:

Certificate in Process Control and Automation
Bachelor of Science in Applied Physics & Instrumentation
Bachelor of Acience in Analytical and Pharmaceutical Chemistry
Bachelor of Science (Hons) in Analytical Chemistry with Quality Assurance
Bachelor of Science (Hons) in Environmental Science and Sustainable Technology
Bachelor of Science (Hons) in Instrument Engineering
Physical Sciences (Common Entry)

Programme Code:

SPRCA_6_Y1	SPHYS_7_Y1	SCHEM_7_Y1	SCHQA_8_Y1
SINEN_8_Y1	SESST_8_Y1	SOMNI_8_Y1	

External Examiner: **Dr James Cruickshank**
Internal Examiner: **Ms. M. Brennan**

Instructions: **Answer Question 1 (40 marks) and TWO other questions (30 marks each).**

Duration: **2 HOURS**
Sitting: Autumn 2018
Requirements for this examination: Mathematical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

1. **COMPULSORY**

(a) Differentiate $f(x) = -4x^2 + 3x - 6$ from first principles. (5 marks)

(b) Find the equation of the tangent to the curve $f(x) = \dfrac{x-2}{x+3}$ at the point is $(-2, -1)$. (5 marks)

(c) The temperature $\theta(t)$ ($°C$) of a body at time t seconds is given by $\theta(t) = 15 + 35e^{-0.3t}$. Find the rate at which the temperature is changing when $t = 10$ seconds. (5 marks)

(d) Sketch the graph of a function $f(x)$ with all the following properties: (1,2), (3,4) and (5,6) lie on the graph; $f'(1) = 0$ and $f'(5) = 0$; $f''(x) > 0$ for $x < 3$, $f''(3) = 0$, and $f''(x) < 0$ for $x > 3$. (5 marks)

(e) Evaluate the indefinite integral $\displaystyle\int \frac{3y}{\sqrt{4y^2+9}}\, dy$. (5 marks)

(f) Evaluate the definite integral $\displaystyle\int_2^4 \left(x^2 + \frac{2}{x^2} - \frac{1}{\sqrt{x}}\right) dx$. (5 marks)

(g) The acceleration a (ms^{-2}) of an object is given by $a(t) = 5e^{-0.2t}$ where t is the time (seconds). Derive expressions for the velocity v (ms^{-1}) and the displacement x (m) at time t (s) if the initial velocity and displacement are both zero. (5 marks)

(h) Find the root mean square value of $y = x + \dfrac{1}{x}$ over [1,2]. (5 marks)

2. (a) Differentiate each of the following with respect to the given variable.

(i) $y = \dfrac{6}{x} + \ln x - \cos(2x) + 1.4$ (4 marks)

(ii) $y = t^3 \cos(e^{-7t})$ (5 marks)

(iii) $y = \dfrac{\sin(2\pi x + 0.4)}{\sqrt{5 + x}}$ (5 marks)

(b) Let $y = -3x^2 + 12x - 6$.

(i) Find the slope of the curve at $x = 5$.

(ii) Find the point at which the tangent at $x = 5$ crosses the x-axis. (8 marks)

(c) Find the dimensions of the rectangular garden of greatest area that can be fenced off (all four sides) with 240 metres of fencing. (8 marks)

3. (a) Evaluate each of the following integrals:

(i) $\displaystyle\int \left(4 + 6x^{-2} + \frac{1}{e^{7x}} + \sin x\right) dx$ (5 marks)

(ii) $\displaystyle\int_0^2 \left(\frac{e^{-6t} + e^{6t}}{3}\right) dt$ (5 marks)

(iii) $\displaystyle\int \frac{t^5}{(1 + t^6)^3}\, dt$ (5 marks)

(b) Find the volume of the solid of revolution generated by revolving about the x-axis the region under the graph of $y = 3x + 2$ from $x = 0$ to $x = 1$. (6 marks)

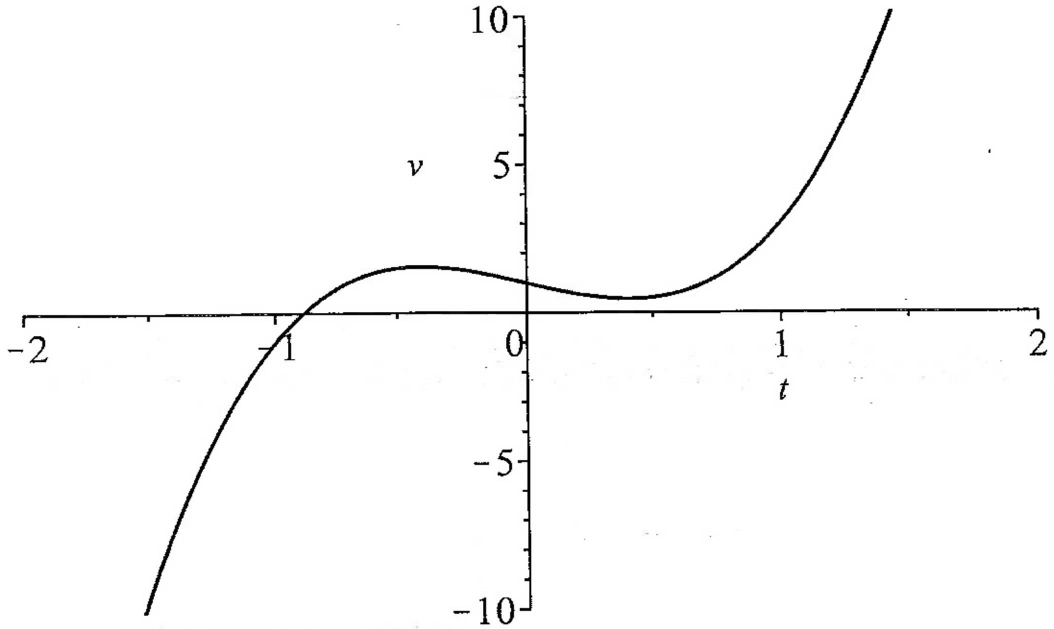
(c) Let $f(x) = 10 - x^2$ and $g(x) = x^2 + 2$.

(i) Find the points of intersection of $f(x)$ and $g(x)$.

(ii) Draw the graphs of $f(x)$ and $g(x)$, shading in the area enclosed by them.

(iii) Use integration to find the shaded area enclosed by the curves $f(x)$ and $g(x)$. (9 marks)

4. (a) The graph of the velocity function $v(t) = 1-2t+4t^3$ is shown below. Use the Newton-Raphson method to find the time at which the velocity is zero correct to 3 decimal places. (12 marks)



(b) Find and classify the turning points on the curve $y = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 12x$. Hence sketch the graph of y. (10 marks)

(c) Water flows into a parallel sided tank at a constant rate and out through a valve which acts as a linear flow resistance. Under these circumstances the differential equation describing the level $h(t)$ metres of water at any time t seconds is described by the differential equation

$$20\frac{dh}{dt} + h = 0.5$$

(i) Solve the equation subject to the initial condition $h(0) = 0$ metres.

(ii) State the approximate time taken for the level to reach its steady state value.

(iii) Sketch the solution. (8 marks)