CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Repeat Semester 2 Examinations 2017/18

Module Title: Mathematics for Science 2.2 with Maple

Module Code :	**MATH6038**
School :	Science and Informatics
Programme Title :	Bachelor of Science in Applied Physics and Instrumentation – Year 2 Higher Certificate in Industrial Measurement and Control – Year 2
Programme Code :	SPHYS_7_Y2 SIMCT_6_Y2
External Examiner :	Dr. A. O' Shea
Internal Examiners :	Dr. M. Brennan Dr. J. P. McCarthy
Instructions :	Answer Q1(compulsory) and 2 other questions.
Duration :	2 Hours
Sitting :	Autumn 2018
Requirements :	Mathematical Tables

Q1. (a) Determine A^{-1} where $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & -4 \\ 2 & 4 & -3 \end{bmatrix}$.

(10 marks)

(b) A component is defective if it is oversized. A sample of 460 components produced by a machine have a mean size of 7.2 cm and a standard deviation of 0.12 cm. The maximum size of an acceptable component is 7.46 cm. Assume a *normal distribution.*

(i) Determine how many components are defective.

(ii) Find how many components are acceptable.

(8 marks)

(c) On the way home from work Pat rides his bike through a traffic light and then passes over a level crossing. The probability that Pat is stopped at the traffic light is $\frac{2}{3}$ while the probability that he is stopped at the level crossing further up the road is $\frac{1}{5}$.

(i) Represent this information in a tree diagram.

(ii) Find the probability that Pat stops once on his journey home.

(iii) Find the probability that Pat gets home without any stopping.

(9 marks)

(d) Find the 95% confidence interval for the population mean from the sample $\{24.7, 21.8, 23.1, 22.7\}$. Comment briefly on your answer.

(8 Marks)

(e) The probability that a car will not develop a major fault within the first 3 years of its life is 0.997. Assuming a *binomial distribution*, find the probability that out of 20 cars selected at random,

(i) 19 will not develop any major faults in the first 3 years,

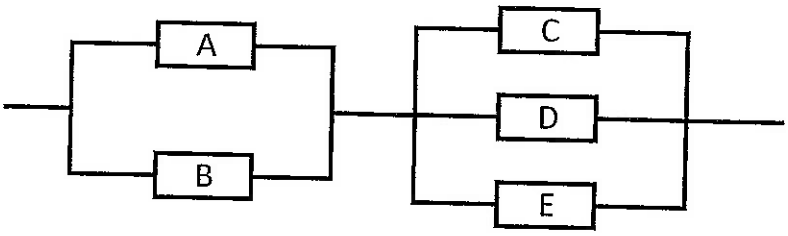
(ii) 19 or more will not develop any major faults in the first 3 years.

(8 marks)

Q1. (e) A *Reliability Block Diagram* is given for a system S where the reliabilities P(A)=0.8, P(B)=0.9 P(C)=0.7,P(D)=0.6 and P(E)=0.9.

Determine the overall systems reliability, carefully showing all steps and intermediate calculations.

(7 marks)



Q2. (a) A corporation wants to lease a fleet of 14 airplanes with a combined carrying capacity of 260 passengers. The three available types of planes carry 10,15 and 20 passengers, respectively.

(i) Identify the variables x, y and z.

(ii) Write down the corresponding *linear system* and find the system's *solution set* S including the real parameter $t \in \mathcal{R}$.

(iii) Find how many of each type of plane could be leased by finding *all positive* solutions.

(16 marks)

(b) The probability that a hard drive fails in any month is 0.004. An engineer is responsible for 500 drives. Assuming a *Poisson distribution*, calculate the probability that in a month the number of drives failing is

(i) none,

(ii) one,

(iii) more than two.

(9 marks)

Q3. A sample of 60 ball bearings was taken from the production of machine A and their diameters (in *cm*) where measured to give the following distribution.

x	0.50-0.55	0.55-0.60	0.60-0.65	0.65-0.70	0.70-0.75	0.75-0.80	0.80-0.85
f	3	8	10	20	14	4	1

(a) Draw an *ogive* and hence determine the *interquartile range.*

(8 marks)

(b) Set up a table indicating the *class marks.* Use ONLY the *assumed mean method* to determine the *mean* and *standard deviation.*

(8 marks)

(c) Find the *mode* for the above distribution.

(5 marks)

(d) Compute the *coefficient of variation* for machine A. Find *Pearson's Coefficient of Skewness* for the above distribution.

(4 marks)

Q4. (a) In order to monitor the quality of a production run of aluminium bolts, 8 samples, each containing 4 components, are taken at random and their diameter lengths are measured correct to the nearest 0.1mm and tabulated as follows:

Sample	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	79.6	82.2	79.7	79.3	81.1	81.3	81.8	83.2
	79.4	80.4	80.1	79.4	81.0	79.9	81.8	80.1
	81.9	81.4	82.5	80.7	82.6	79.8	80.7	77.3
	80.7	81.2	80.7	79.8	81.3	80.2	81.9	79.3
Means $\overline{x}_i$	80.40	81.30	80.75	79.8	81.50	80.30	$\overline{x}_7$	$\overline{x}_8$
Ranges w_i	2.5	1.8	2.8	1.4	1.6	1.5	w_7	w_8

Calculate the remaining sample means $\overline{x}_7$ and $\overline{x}_8$ and ranges w_7 and w_8. Find the grand mean $\overline{\overline{x}}$ and the associated *outer* and *inner control limits.* Hence set up a control chart for the sample means. State, giving reasons, whether or not the process is under control.

(16 Marks)

(b) Using only the *Gauss Jordan Algorithm*, determine the solution set S for the following linear system of equations,

$$\begin{aligned} \frac{1}{4}x \; - \; \frac{3}{2}y &= 1 \\ -2x \; + \; 12y &= -8. \end{aligned}$$

(9 marks)

Statistical Formulae

Population & sample means:

$$\mu,\ \bar{x} = \frac{\sum fx}{\sum f} = A + c\frac{\sum fd}{\sum f}, \qquad \text{where } d = \frac{x-A}{c}$$

Population standard deviation

$$\sigma = \sqrt{\frac{\sum f(x-\mu)^2}{\sum f}} = c\sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

Sample standard deviation:

$$s = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f - 1}} = c\sqrt{\frac{\sum fd^2}{\sum f - 1} - \frac{\sum f}{\sum f - 1}\left(\frac{\sum fd}{\sum f}\right)^2}$$

$$\text{Mode} = L_M + c\left(\frac{\ell}{\ell + u}\right)$$

Coefficient of Variation:

$$\text{CoV} = \frac{\sigma}{\mu} \times 100, \qquad \frac{s}{\bar{x}} \times 100$$

Pearson's Coefficient of Skewness:

$$\text{PSK} = \frac{\text{Mean} - \text{Mode}}{\sigma}, \qquad \frac{\text{Mean} - \text{Mode}}{s}$$

Binomial distribution:

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Poisson distribution:

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!} = e^{-\lambda} \cdot \frac{\lambda^r}{r!}$$

Normal distribution:

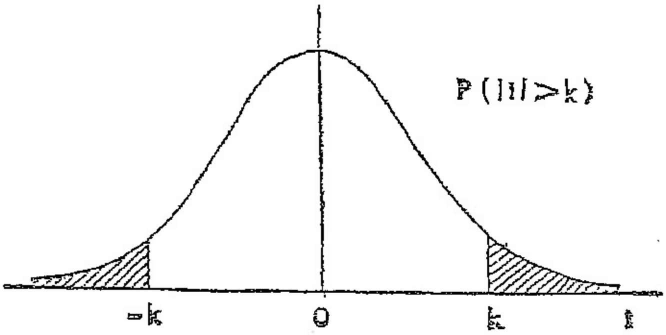
$$z = \frac{X - \mu}{\sigma}$$

Confidence Interval:

$$I = \left[\bar{x} - t_c\frac{s}{\sqrt{n}}, \bar{x} + t_c\frac{s}{\sqrt{n}}\right]$$

t-DÁILEADH t-DISTRIBUTION

n − 1	20	10	5	2	1	0·2
1	3·078	6·314	12·706	31·821	63·657	318·310
2	1·886	2·920	4·303	6·965	9·925	22·327
3	1·638	2·353	3·182	4·541	5·841	10·215
4	1·533	2·132	2·776	3·747	4·604	7·173
5	1·476	2·015	2·571	3·365	4·032	5·893
6	1·440	1·943	2·447	3·143	3·707	5·208
7	1·415	1·895	2·365	2·998	3·499	4·785
8	1·397	1·860	2·306	2·896	3·355	4·501
9	1·383	1·833	2·262	2·821	3·250	4·297
10	1·372	1·812	2·228	2·764	3·169	4·144
11	1·363	1·796	2·201	2·718	3·106	4·025
12	1·356	1·782	2·179	2·681	3·055	3·930
13	1·350	1·771	2·160	2·650	3·012	3·852
14	1·345	1·761	2·145	2·624	2·977	3·787
15	1·341	1·753	2·131	2·602	2·947	3·733
16	1·337	1·746	2·120	2·583	2·921	3·686
17	1·333	1·740	2·110	2·567	2·898	3·646
18	1·330	1·734	2·101	2·552	2·878	3·610
19	1·328	1·729	2·093	2·539	2·861	3·579
20	1·325	1·725	2·086	2·528	2·845	3·552
21	1·323	1·721	2·080	2·518	2·831	3·527
22	1·321	1·717	2·074	2·508	2·819	3·505
23	1·319	1·714	2·069	2·500	2·807	3·485
24	1·318	1·711	2·064	2·492	2·797	3·467
25	1·316	1·708	2·060	2·485	2·787	3·450
26	1·315	1·706	2·056	2·479	2·779	3·435
27	1·314	1·703	2·052	2·473	2·771	3·421
28	1·313	1·701	2·048	2·467	2·763	3·408
29	1·311	1·699	2·045	2·462	2·756	3·396
30	1·310	1·697	2·042	2·457	2·750	3·385
40	1·303	1·684	2·021	2·423	2·704	3·307
60	1·296	1·671	2·000	2·390	2·660	3·232
120	1·289	1·658	1·980	2·358	2·617	3·160
∞	1·282	1·645	1·960	2·326	2·576	3·090



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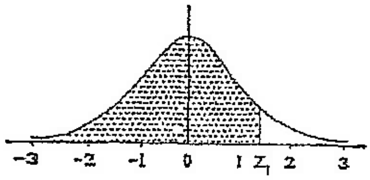
Area under the Normal Curve

$$P(z \leqslant z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2}\, dz .$$



z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0·1	0·5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
0·2	0·5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
0·3	0·6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
0·4	0·6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0·5	0·6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0·6	0·7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0·7	0·7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
0·8	0·7881	7910	7939	7967	7995	8023	8051	8078	8106	8133
0·9	0·8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1·0	0·8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1·1	0·8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1·2	0·8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1·3	0·9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1·4	0·9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1·5	0·9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1·6	0·9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1·7	0·9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1·8	0·9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1·9	0·9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2·0	0·9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2·1	0·9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2·2	0·9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2·3	0·9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2·4	0·9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2·5	0·99379	99396	99413	99430	99446	99461	99477	99492	99506	99520
2·6	0·99534	99547	99560	99573	99585	99598	99609	99621	99632	99643
2·7	0·99653	99664	99674	99683	99693	99702	99711	99720	99728	99736
2·8	0·99744	99752	99760	99767	99774	99781	99788	99795	99801	99807
2·9	0·99813	99819	99825	99831	99836	99841	99846	99851	99856	99861
3·0	0·99865	99869	99874	99878	99882	99886	99889	99893	99897	99900
3·1	0·99903	99906	99910	99913	99916	99918	99921	99924	99926	99929
3·2	0·99931	99934	99936	99938	99940	99942	99944	99946	99948	99950
3·3	0·99952	99953	99955	99957	99958	99960	99961	99962	99964	99965
3·4	0·99966	99968	99969	99970	99971	99972	99973	99974	99975	99976
3·5	0·99977	99978	99978	99979	99980	99981	99981	99982	99983	99983
3·6	0·99984	99985	99985	99986	99986	99987	99987	99988	99988	99989
3·7	0·99989	99990	99990	99990	99991	99991	99992	99992	99992	99992
3·8	0·99993	99993	99993	99994	99994	99994	99994	99995	99995	99995
3·9	0·99995	99995	99996	99996	99996	99996	99996	99996	99997	99997

Control Chart Coefficients

Table 1

n	2	3	4	5	6	7	8	9
a_n	0.8862	0.5908	0.4857	0.4299	0.3946	0.3698	0.3512	0.3367

Table 2

Sample Size n	2	3	4	5	6	7	8	9	10	11	12
$A_{0.025}$	1.229	0.608	0.476	0.377	0.316	0.274	0.244	0.202	0.220	0.186	0.174
$A_{0.001}$	1.937	1.054	0.750	0.594	0.498	0.432	0.384	0.347	0.317	0.294	0.274

Table 3

	For use with σ				For use with $\overline{w}$			
n	$D_{0.001}$	$F_{0.025}$	$F_{0.975}$	$D_{0.999}$	$D'_{0.001}$	$F'_{0.025}$	$F'_{0.975}$	$D'_{0.999}$
2	0.00	0.04	3.17	4.65	0.00	0.04	2.81	4.12
3	0.06	0.30	3.68	5.06	0.04	0.18	2.17	2.99
4	0.20	0.30	3.98	5.31	0.10	0.29	1.93	2.58
5	0.37	0.85	4.20	5.48	0.16	0.37	1.81	2.36
6	0.54	1.06	4.36	5.62	0.20	0.42	1.72	2.22
7	0.69	1.25	4.49	5.73	0.26	0.46	1.66	2.12
8	0.83	1.41	4.61	5.82	0.29	0.50	1.62	2.04
9	0.96	1.55	4.70	5.90	0.32	0.52	1.58	1.99
10	1.08	1.67	4.79	5.97	0.35	0.54	1.56	1.94
11	1.20	1.78	4.86	6.04	0.38	0.56	1.53	1.90
12	1.30	1.88	4.92	6.09	0.40	0.58	1.51	1.87