CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/18

Module Title: Technological Maths 201

Module Code:	**MATH6040**
School:	School of Civil, Structural and Environmental Engineering School of Mechanical, Electrical & Process Engineering
Programme Titles:	B Eng (Hons) in Sustainable Energy Engineering – Year 2 Bachelor of Engineering in Biomedical Engineering – Year 2 Bachelor of Engineering in Civil Engineering– Year 2 Bachelor of Engineering in Environmental Engineering– Year 2 Bachelor of Engineering in Manufacturing Engineering – Year 2 Bachelor of Engineering in Mechanical Engineering – Year 2 Certificate in Environmental and Energy Engineering – Year 1 Mechanical Science – Year 1 Part-time BEng in Mechanical Engineering – Year 2
Programme Codes:	ESENT_8_Y2 EBIME_7_Y2 CCIVL_7_Y2 CENVI_7_Y2 EMANF_7_Y2 EMECH_7_Y2 EENEN_7_Y1 EMSCI_6_Y1 EMECN_7_Y2
External Examiner:	Dr James Cruickshank
Internal Examiners:	Ms J. English , Dr J.P. McCarthy, Ms S. Murphy
Instructions:	Answer ALL questions.
Duration:	2 HOURS
Sitting:	Autumn 2018
Exam Requirements:	Mathematics Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.

If in doubt please contact an Invigilator.

1. (a) Given the vectors

$$\mathbf{u} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\,, \text{ and } \qquad \mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k},$$

i. determine the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$.

ii. show that the vectors $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are perpendicular.

[7 Marks]

(b) If the vector $\mathbf{w} = \overrightarrow{PQ}$, where P is the point $(1, 3, 2)$ and Q is the point $(-1, 0, 8)$:

i. express the vector $\mathbf{w}$ in Cartesian form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.

ii. find a vector of magnitude 11 in the direction *opposite* to that of $\mathbf{w}$.

[5 Marks]

(c) If forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are defined by

$$\mathbf{F}_1 = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\,, \text{ and } \qquad \mathbf{F}_2 = -3\mathbf{i} - 2\mathbf{j},$$

find the work done by the resultant force $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$ in moving a particle from the point $A(1, 1, 1)$ to the point $B(4, -1, 2)$. Assume the force is measured in kN and the distance in metres.

[6 Marks]

(d) A force of magnitude 18 kN in the direction of $4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ is applied at the point $P(1, -1, 2)$.

i. Find the moment of the force about the point $Q(2, -1, 3)$.

ii. Find, also, the magnitude of this moment.

Assume distance is measured in metres.

[7 Marks]

2. (a) Given the matrices

$$A = \begin{pmatrix} 4 & -3 \\ 6 & -4 \\ 10 & 0 \end{pmatrix}, B = \begin{pmatrix} 9 & -7 \\ 9 & -3 \end{pmatrix}, C = \begin{pmatrix} 3 & -11 & 10 \end{pmatrix}, D = \begin{pmatrix} 8 & -5 & 2 \\ -9 & 6 & -7 \end{pmatrix},$$

determine the following (if defined).

$$(i)\, A + D^T \qquad (ii)\, CA \qquad (iii)\, C^2 \qquad (iv)\, B^{-1}$$

[7 Marks]

(b) Does the following matrix have an inverse? Justify your answer.

$$\begin{pmatrix} 5 & -1 & 0 \\ 6 & 2 & -3 \\ 1 & 3 & -3 \end{pmatrix}$$

[5 Marks]

(c) An engineer analysed an electrical circuit using Kirchhoff's Laws and found that the currents x, y and z, measured in amps, satisfied:

$$\begin{aligned} x - y - z &= 0 \\ 2x + 5z &= 6 \\ 2x + 4y &= 4 \end{aligned}$$

i. Write the linear system as a matrix equation $A\mathbf{x} = \mathbf{b}$.

[3 Marks]

ii. If $A\mathbf{x} = \mathbf{b}$ find $\mathbf{x}$ in terms of A and $\mathbf{b}$.

[2 Marks]

iii. It can be shown that

$$\begin{pmatrix} 1 & -1 & -1 \\ 2 & 0 & 5 \\ 2 & 4 & 0 \end{pmatrix}^{-1} = \frac{1}{38} \begin{pmatrix} 20 & 4 & 5 \\ -10 & -2 & 7 \\ -8 & 6 & -2 \end{pmatrix}.$$

Hence, or using Cramer's Rule, solve the linear system.

[7 Marks]

3. (a) A curve is defined by parametric equations $x = 4t + 2$ and $y = 6t^2 - 4$.

   i. Find $\dfrac{dy}{dx}$ at the point $(6, 2)$.

   ii. Find $\dfrac{d^2y}{dx^2}$.

   iii. Find the point where the slope is equal to 4.5.

   [7 Marks]

   (b) Consider the curve described by

   $$x^2\, y^2 - 4x^3 - 5y^2 = 0.$$

   Find the equation of the tangent to this curve at the point $(-1, 1)$.

   [10 Marks]

   (c) The surface area of a spherical balloon is increasing at a rate of 12 cm^2/s. How fast is the volume of balloon increasing when the radius equals 3 cm?

   $$\left[\text{HINT:} \qquad A = 4\pi r^2\,, \qquad V = \frac{4}{3}\pi r^3\right].$$

   [8 Marks]

4. (a) Find:

$$\int \frac{1}{2x^2 + 8x + 40}\, dx.$$

[7 Marks]

(b) A force (in N) acting on an object is described by

$$F(x) = \ln x$$

where x is the displacement (in m) of the object from a point O.

The object is moved from a point 1 m from O to a point 20 m from O. Determine the work done by the force, stating units.

[7 Marks]

(c) Consider the region below the curve $y = e^{-x}$ between the ordinates $x = 0$ and $x = 1$.

i. Draw a rough sketch of the region.
ii. Find the area of this region.
iii. Find the coordinates of the centroid of this region.
iv. Indicate on your sketch from part i. the position of the centroid.

[11 Marks]

Appendix: Some Formulae

Centroid: $\bar{x} = \dfrac{\int xy\, dx}{\int y\, dx}\,;\ \ \bar{y} = \dfrac{\frac{1}{2}\int y^2\, dx}{\int y\, dx}.$

Volume: $V = \pi \int y^2\, dx.$

Centre of Gravity: $\bar{x} = \dfrac{\int xy^2\, dx}{\int y^2\, dx}.$

Work: $W = \int F\, ds.$

Work (expanding gas): $W = \int P\, dV.$