CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Repeat Semester 2 Examinations 2017/18

Module Title: Technological Mathematics 221

Module Code : **MATH6043**

Schools : Mechanical, Electrical & Process Engineering/Science & Informatics

Programme Title :
Bachelor of Engineering (Honours) in Electrical Power Systems-II
Bachelor of Engineering (Honours) in Electronic Systems Engineering-II
Bachelor of Science (Honours) in Instrument Engineering-II
Bachelor of Engineering in Electrical Engineering-II
Bachelor of Engineering in Electronic Engineering-II

Programme Code :
EEPSY_8_Y2
EELES_8_Y2
SINEN_8_Y2
EELEC_7_Y2
EELXE_7_Y2

External Examiner : Dr. J. Cruickshank

Internal Examiners : Dr. M. Brennan

Instructions : Answer Q1(COMPULSORY) and any other 2 questions.
Q1 is worth 50 marks while all other questions are worth 25 marks.

Duration : 2 Hours

Sitting : Autumn 2018

Requirements : Mathematical Tables

Q1. (a) Use *separation of variables* to solve the *initial value problem explicitly* for y,

$$\frac{dy}{dx} = \frac{4x^2y^2}{x^3+1}, \qquad y(0) = 3.$$

(13 marks)

(b) Solve the *initial-value* problem for y,

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 8y = 0, \qquad y(0) = 4, y'(0) = 2.$$

(10 marks)

(c) The number of power surges at an electrical plant satifies a *Poisson distribution* with an average of 0.4 power surges per day. Find the probability that

(i) no power surges will occur on a given day.

(ii) no more than 2 power surges will occur on a given day.

(8 marks)

(d) The rate of flow X of a gas in a wind turbine follows a *normal distribution* with mean $3\,m^3/s$ and standard deviation of $0.015\,m^3/s$. Find the probability that the flow rate is recorded as being between $2.96\,m^3/s$ and $3.04\,m^3/s$

(9 marks)

(e) Solve the *homogeneous second order differential equation*,

$$y'' + 4y' + 13y = 0.$$

Hence write down a *best guess* for y_P, the particular solution to

$$y'' + 4y' + 13y = 3e^{-2x}\cos 3x + 12$$

Do NOT solve for the constants in y_P.

(10 marks)

Q2. (a) Find the inverse of the matrix $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & -2 & -1 \\ 3 & 1 & 2 \end{bmatrix}$.

Hence solve the following system using *only* the *inverse matrix method.*

$$\begin{aligned} 4x &- y &+ 6z &= 25 \\ 2x &- 2y &- z &= 6 \\ 3x &+ y &+ 2z &= 12. \end{aligned}$$

(15 marks)

(b) A switching system contains three switches, X, Y and Z of different manufacture that are independent of each other. The probability that they fail within a year are 0.2,0.35 and 0.45 respectively. Calculate the probability that within a year of installation

(i) Only switch Z will fail;

(ii) Two switches fail;

(iii) At least one of the switches will fail.

(10 marks)

Q3. (a) Use *Method of Undetermined Coefficients* to find the general solution to the following *nonhomogeneous* second order differential equation.

$$2\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 5y = 10x^2 - 7x + 4.$$

(15 marks)

(b) *GPS PatPat* watches are reported to have a return rate of 28% due to various technical faults. In a batch of 12 watches, apply the *binomial distribution* to determine the probability that;

(i) exactly 3 watches are returned,

(ii) at most 2 watches are returned.

(10 marks)

Q4. (a) A machine produces resistors of mean resistance $470k\Omega$ and standard deviation $10.2k\Omega$. A resistor is chosen at random. Assuming a *normal distribution*, find the probability that the resistor has resistance

(i) greater than $475k\Omega$,

(ii) between $457k\Omega$ and $482k\Omega$.

Determine α if 95% of the production lies in the range $(470 \pm \alpha)k\Omega$.

(15 marks)

(b) A system S consists of five independent components in series, each having the same reliability of 0.970.

(i) What is the overall reliability of the system S?

(ii) Let W be where an additional five *similar* components are all added to create a 10-component system in series. Determine the reliability of system W.

(iii) Find the necessary increase in each individual component, in order for the system W overall reliability to be the same as the original system S.

(10 marks)

Probability Distributions Formulae

Binomial Distribution:

$$P(X = r) = {}^nC_r \, p^r q^{n-r}, \qquad \text{mean} = np; \qquad \text{variance} = npq$$

Poisson Distribution:

$$P(X = r) = \frac{\lambda^r . e^{-\lambda}}{r!}, \qquad \lambda = \text{mean} = np$$

Standard Normal Units:

$$X \sim N\{\mu, \sigma\} \Rightarrow Z = \frac{X - \mu}{\sigma}$$