Cork Institute of Technology

Institiúid Teicneolaíochta Chorcaí

Autumn Examinations 2017/2018

Module Title: Mathematics for Computer Science

Module Code:	MATH6055
School:	School of Computer Science
Programme Title:	Bachelor of Science (Honours) in Software Development – Year 1 Bachelor of Science (Honours) in Computer Systems – Year 1 Bachelor of Science (Honours) in IT Management – Year 1 Bachelor of Science (Honours) in Web Development – Year 1 Bachelor of Science in Information Technology – Year 1 Bachelor of Science in Software Development – Year 1 Higher Certificate in Software Development – Year 1
Programme Code:	CR_KSDEV_8 CR_KDNET_8 CR_KITMN_8 CR_KWEBD_8 CR_KITSP_7 CR_KCOMP_7 CR_KCOMP_6
External Examiners(s):	Dr. James Cruickshank
Internal Examiners(s):	Dr. R. Heffernan, Dr. J.P. McCarthy, Dr. M. Nicholson Mr. A. Naeem and Mr. A. O'Connor
Instructions:	Answer all four questions. Each question carries 25 marks.
Duration:	2 Hours
Sitting:	Autumn 2017
Requirements for this examination:	Mathematical Tables

Note to Candidates:

Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

Question 1.

(a) i. Write the following expression as a single fraction

$$\frac{x}{3+x} - \frac{y}{2}$$

[3 Marks]

ii. Factor the following as much as possible

$$x^3 + 3x^2 - 4x$$

[3 Marks]

iii. Solve for a in the following equation

$$\frac{a+x}{b-x} = c$$

[3 Marks]

iv. Write

$$\frac{a}{a^4}\left(\frac{1}{a^2}\right)^3$$

in the form a^p , where p is a rational number. **[5 Marks]**

(b) Solve for x in the following equations.

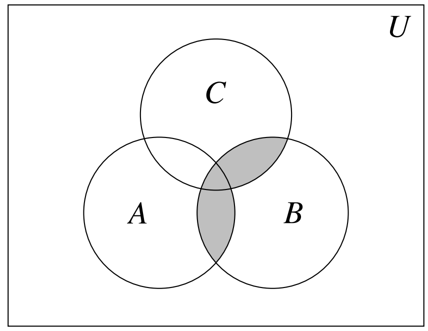
i. $\log_3(x+1) - \log_3(x-1) = 3\log_3(2)$ **[5 Marks]**

ii. $5 = 3e^{2x}$ **[3 Marks]**

(c) How many (binary) bits are necessary to represent 9834 distinct numbers? **[3 Marks]**

Question 2.

(a) Use symbols to describe the shaded area in the following Venn diagram.



[4 Marks]

(b) Let $A = \{a, b, c, d, e\}$ and $a\text{R}b$ if and only if both a and b are both vowels or both consonants. List the elements of the relation R.

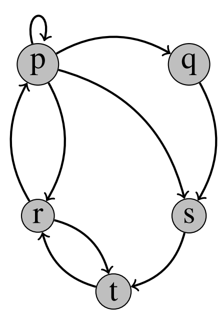
[3 Marks]

(c) Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ and $X = \{a, b\}$.
List the elements of $(A \cap B) \times X$ and the elements of $(A \times X) \cap (B \times X)$. What can you conclude?

[6 Marks]

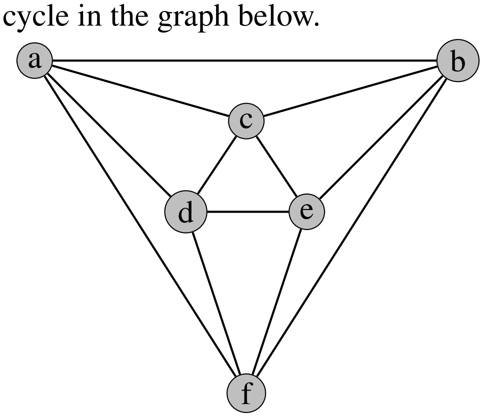
(d) List the elements of V and E, where V is the set of vertices and E is the set of edges of the graph represented in the figure below.

[4 Marks]



(e) i. State a condition that guarantees that a graph with n vertices will have a Hamilton cycle. **[3 Marks]**

ii. Find a Hamilton cycle in the graph below. **[5 Marks]**



Question 3.

(a) Let $X = \{-1, 0, 1, 2\}$ and $Y = \{-4, -2, 0, 2\}$. Define the function $f : X \to Y$ as $f(x) = x^2 - x$. Is the function f invertible?

[4 Marks]

(b) Determine which of the following functions are onto:

i. $f_1 : \mathbb{R} \to \mathbb{R}$ where $f_1(x) = x^2 - 1$.

ii. $f_2 : \mathbb{Z} \to \mathbb{Z}$ where $f_2(n) = n^3$.

iii. $f_3 : \mathbb{R} \to \mathbb{R}$ where $f_3(x) = x^3$.

[7 Marks]

(c) Let $X = \{1, 2, 3, 4\}$. Let $f : X \to \mathbb{R}$ be a function defined as the set of ordered pairs $\{(1,2), (2,3), (3,4), (4,5)\}$. Let $g : \mathbb{R} \to \mathbb{R}$ be the function defined as $g(x) = x^2$. List the ordered pairs of $g \circ f$.

[4 Marks]

(d) Produce rough sketches of the following lines (you may plot a number of lines on a single plane):

i. $\ell_1(x) = 2x$

ii. $\ell_2(x) = -3x$

iii. $\ell_3(x) = 2x + 3$

iv. $\ell_4(x) = -4x + 2$ and

v. $\ell_5(x) = -4x - 3$.

[5 Marks]

(e) List the following list of functions $\mathbb{R}_+ \to \mathbb{R}$ from slowest growing to fastest growing:

$$2x^2,\ 2e^{3x},\ 4\ln x,\ 2x \text{ and } x^{10}.$$

Here $\mathbb{R}_+$ is the set of all *positive* real numbers, i.e. $\mathbb{R}_+ = (0, \infty)$. **[5 Marks]**

Question 4.

(a) Consider the following recurrence relation

$$P(n) = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot P(n-1) & \text{if } n > 0 \end{cases}$$

i. Compute $P(4)$. **[4 Marks]**

ii. Write $P(1000)$ in terms of $P(998)$. **[3 Marks]**

(b) Consider the following sequence

$$6, 11, 16, 21, 26, 31, 36, \ldots$$

i. Give a recurrence relation that describes this sequence. **[4 Marks]**

ii. Is this a arithmetic or geometric sequence? **[2 Marks]**

iii. Sum the first 50 terms of this sequence. **[4 Marks]**

(c) Suppose that today (year 0) your car is worth €15,000. Each year your car looses 10% of its value, but at the end of each year you add customisations to your car which increase it's value by €50.

i. What will be the value of your car after 1 year? **[2 Marks]**

ii. Write a recurrence relation to model the situation. **[6 Marks]**