CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2018

Module Title: Mathematics for Biological Sciences

Module Code: **MATH 6056**

School: School of Science and Informatics.

Programme Title:

BSc in Applied Biosciences – Year 1
Common Entry Biological Sciences – Year 1
BSc in Nutrition and Health Science – Year 1
BSc in Pharmaceutical Biotechnology – Year 1
BSc in Herbal Science – Year 1

Programme Code:

SBIOS_7_Y1
SCEBS_8_Y1
SNHSC_8_Y1
SPHB_8_Y1
SHERB_8_Y1

External Examiner: **Dr. James Cruickshank.**
Internal Examiner: **H Lordan, J McGuinness, S Murphy, M Quirke**

Instructions: Answer **ALL** four questions.
Show all calculations in full.

Duration: **2 HOURS**

Sitting: **Autumn 2018**

Requirements for this examination: Graph paper, Formulae & Tables Book

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt, please contact an Invigilator.

Question 1

(a) The molar heat capacity of a solid compound is given by

$$c = a + bT$$

(i) When $c = 52.4$, $T = 99.8$ and when $c = 171.6$, $T = 397.8$. Using this information formulate two equations.

(ii) Using the equations you found in part (i), find the values of a and b.

(iii) Verify your answer.

[9 marks]

(b) A solution, containing water as the solvent, has a concentration of 0.75 mg/mL.

(i) What is this concentration in ppm (parts per million)?

(ii) What is this concentration in ppb (parts per billion)?

[7 marks]

(c) In a lab, you are given a cube of pure copper. You measure the sides of the cube to find the volume and weigh it to find its mass.

(i) If the volume was found to be 2.86cm^3 and the mass was found to be 25,120 mg, find the density (g/cm^3) of the cube of copper, correct to two decimal places using the following formula:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

(ii) Given that the accepted density of copper is 8.96 g/cm^3, calculate the relative percentage error in the density calculation that you found in part (i), correct to two decimal places.

[9 marks]

Question 2

(a) Simplify the following expression and write your answer using positive indices only:

$$\left(\frac{2\,a^2\,b^4\,c^{-1}}{3\,a\,b^2c}\right)^2$$

[5 marks]

(b) Solve for x in each of the following

(i) $3^{5x+4} = \frac{1}{9^{4-x}}$

(ii) $\log_5(10x + 15) - \log_5(2x - 1) = 2$

[10 marks]

(c) The radioactive decay of Carbon-14 is described by

$$N(t) = N_0 e^{-\lambda t}$$

where $N(t)$ is the number of Carbon-14 atoms present in a sample at a time t (years), N_0 is the initial number of atoms present and λ is the decay rate.

Given that $N_0 = 100{,}000\ atoms$ and $\lambda = 0.00012\ yr^{-1}$

(i) Find the number of Carbon-14 atoms present after 100 years. Round your answer to the nearest atom.

(ii) Find how long it will take for half of the sample to decay. Round your answer to the nearest year.

[10 marks]

Question 3

(a) Differentiate each of the following by rule:

(i) $y = 3x^4 - 10\sqrt{x} + 12$

(ii) $y = 4\cos x + \sin x - 3e^x$

[9 marks]

(b) The distance s, (metres) of a moving particle from its starting point is:

$$s = 3 + 8t - 7t^2$$

where t is the time in seconds.

(i) Write down expressions for velocity and acceleration.

(ii) What is the velocity after 3 seconds?

(iii) What is the acceleration after 3 seconds?

[10 marks]

(c) The revenue, R (euro) earned by a chartered coach on a weekend excursion is

$$R(x) = 90x - x^2$$

where x is the number of seats taken in the 60-seater bus.

(i) Find the number of seats sold that will maximise revenue.

(ii) What will this maximum revenue be?

[6 marks]

Question 4

(a) Reduce the following equations to their linear form. Indicate what you would plot along each axis and state clearly the slope and the intercept in each case.

(i) $\frac{1}{F} = ax^2 + b$ where a and b are constants

(ii) The period T of a simple pendulum is related to its length L by the formula

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g and π are constants.

(iii) The mass of a substance is believed to dissolve in one litre of water at temperature, t, according to the law:

$$m = ae^{bt}$$

where a and b are constants.

[15 marks]

(b) The resistance R ohms (Ω) of a copper winding is measured at various temperatures $t\ °C$ and the law relating resistance and temperature is of the form

$$R = at + b$$

where a and b are constants.

The resistance of a copper winding was measured at various temperatures and the resulting data was plotted on a graph. A version of this graph, with points marked in, can be found in figure 1 on page 6. By using this graph

(i) Determine the values of the constants a and b. Give a accurate to 3 decimal places and b to the nearest whole number

(ii) State the linear relationship between resistance of a copper wire and temperature.

(iii) Using the linear relationship you have stated in part (ii), find the resistance given a temperature of $-25°C$.

(iv) Using the linear relationship you have stated in part (ii), find the value of temperature when the resistance is 124 Ω.

[10 marks]

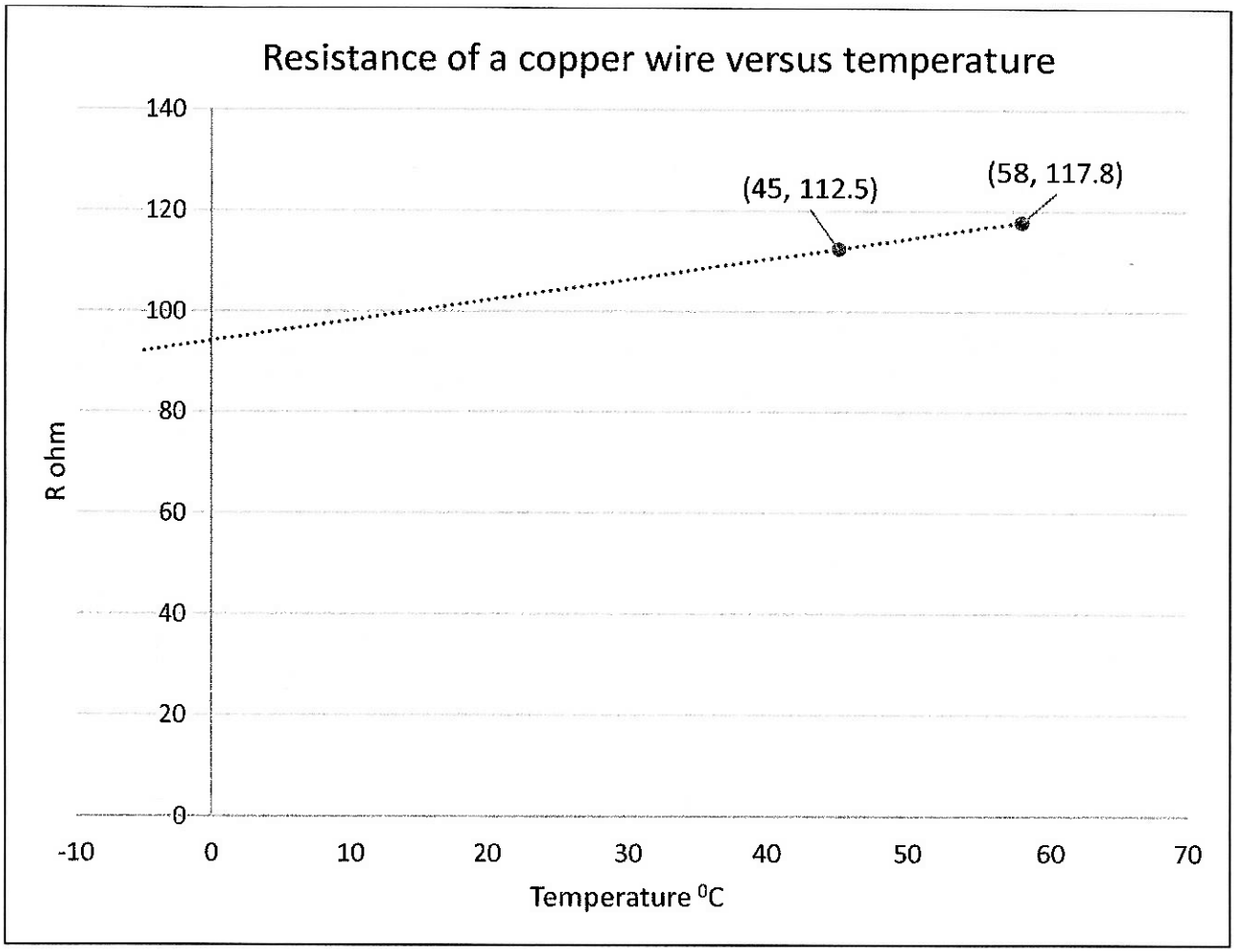

Figure 1: Plot of Resistance of a copper wire versus temperature

Expression of Concentration	Formula
Molarity mol/L or M	$\frac{\text{Moles of solute (mol)}}{\text{Volume of solution (L)}}$
Volume % (vol/vol %)	$\frac{\text{Volume of solute}}{\text{Volume of solution}} \times 100$ **Note**: units need to be consistent
Mass % (mass/mass %)	$\frac{\text{Mass of solute}}{\text{Mass of solution}} \times 100$ **Note**: units need to be consistent
Mass per volume %	$\frac{\text{Mass of solute (g)}}{\text{Volume of solution (mL)}} \times 100$ **Note:** Assumes solvent is water (mixes gram and mL since density of water 1g/mL)
parts per million (ppm)	$\frac{\text{Mass of solute}}{\text{Mass of solution}} \times 10^6$ **Note**: units need to be consistent **Note**: 1 ppm = 1 mg/L (assuming solvent is water)
parts per billion (ppb)	$\frac{\text{Mass of solute}}{\text{Mass of solution}} \times 10^9$ **Note**: units need to be consistent **Note**: 1 ppb = 1 µg/L (assuming solvent is water)

Table 1: Different Units for Expressing the Concentration of Solutions

Further equations for calculating concentration:

$$C_1V_1 = C_2V_2$$

$$density = \frac{mass}{volume}$$

$$Concentration\ (mol/l) = \frac{mass\ (g)}{Volume\ (l)} \times \frac{1}{molecular\ mass\ (g/mol)}$$

The density of water is assumed to be 1 g/cm^3 or 1 g/mL. Therefore 1ml H_20 = 1 g or 1 cm^3 H_2O = 1 g