CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/2018

Module Title: Engineering Maths Methods

Module Code: **MATH7005**

School: School of Mechanical, Biomedical & Manufacturing Engineering

Programme Title: Bachelor of Engineering (Honours) in Mechanical Engineering
Bachelor of Engineering (Honours) in Biomedical Engineering

Programme Code: **EMECH_8_Y2**
EBIOM_8_Y2

External Examiner(s): **Dr Ann O'Shea**

Internal Examiner(s): **Dr Maryna Lishchynska**

Instructions: Answer **ALL** questions.
Show all calculations in full.

Duration: 2 hours

Sitting: Autumn 2018

Requirements for this examination: Tables of Laplace Transforms, Z-transforms, standard integrals and Fourier formulae attached

Note to Candidates: Please check the **Programme Title** and the **Module Title** to ensure that you have received the correct examination.
If in doubt please contact an Invigilator.

Q1.

a) Determine

$$L\{\ 2u(t-3)+2(t-4)u(t-4)+3tu(t-5)\}$$

[4 marks]

b) Find the inverse Laplace transforms of the following expressions:

(i) $\dfrac{96}{\left(s^2+4\right)^2}$

[8 marks]

(ii) $\dfrac{20e^{-3s}}{s^2\left(s^2+4\right)}$

[8 marks]

c) Solve the following differential equations:

(i) $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+3y=10\delta(t-4);\qquad y(0)=y'(0)=0$ **[7 marks]**

(ii) $\dfrac{d^2y}{dt^2}=f(t);\qquad y(0)=y'(0)=0$

where $f(t)=\begin{cases}0 & \text{if}\quad 0\le t<2\\ 4t-8 & \text{if}\quad t\ge 2\end{cases}$ **[8 marks]**

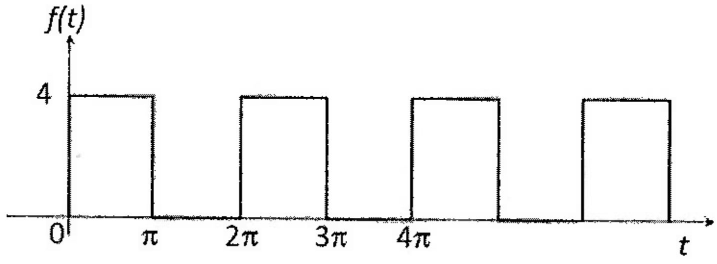
P.T.O.

Q2.

a) A function $f(x)$ is defined as $f(x) = 2 - x$ where $0 \le x \le 2$. Sketch this function and its <u>even and odd</u> extensions. Clearly label each extension on your sketch.

[2 marks]

b) Consider the square wave shown below.



(i) State whether this function is even, odd or neither.

(ii) What is the period of this function?

(iii) Find the Fourier series expansion of this function. Show terms up to 5th harmonics inclusive.

[13 marks]

b) Use Z-transforms to solve the following difference equations:

(i) $y_{n+1} - 4y_n = 12\left(4^n\right)$ $\quad y_0 = 0$ **[5 marks]**

(ii) $y_{n+2} - 4y_{n+1} + 3y_n = 20$ $\quad y_0 = y_1 = 0$ **[8 marks]**

(iii) $y_{n+2} - 2y_{n+1} + y_n = 24$ $\quad y_0 = y_1 = 0$ **[7 marks]**

P.T.O.

Q3.

a) The matrix

$$A = \begin{pmatrix} 5 & 1 & -2 \\ 1 & 5 & -2 \\ -2 & -2 & 4 \end{pmatrix}$$

has the following eigensystem:

$$2\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix};\ 4\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix};\ 8\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

(i) Show that the eigenvectors of matrix A are linearly independent and mutually orthogonal.

(ii) Form an orthogonal matrix P that diagonalises A.

(iii) Explain what it means geometrically to multiply a vector by an orthogonal matrix.

[8 marks]

b) An electrical network with three loops gives rise to the following system of differential equations where $x(t)$, $y(t)$ and $z(t)$ are the currents in each loop correspondingly:

$$\begin{cases} \dfrac{dx}{dt} = 3x + 2y + 2z \\ \dfrac{dy}{dt} = 2x + 3y + 2z \\ \dfrac{dz}{dt} = 2x + 2y + 3z \end{cases}$$

Assume an exponential solution and find the general solution of the system.

[14 marks]

c) In the theory of mechanical vibrations the system of differential equations below arises, where $x_1(t)$ and $x_2(t)$ are the displacements of two masses attached by two springs. Assume periodic solutions of the form $R\cos(\omega t - \alpha)$ and find the general solution of the system:

$$x_1'' = -5x_1 - 11x_2$$
$$x_2'' = -x_1 - 15x_2$$

[8 marks]

Table of Laplace Transforms

$f(t)$	$F(s)$
a	$\frac{a}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin wt$	$\frac{w}{s^2+w^2}$
$\cos wt$	$\frac{s}{s^2+w^2}$
$\sinh kt$	$\frac{k}{s^2-k^2}$
$\cosh kt$	$\frac{s}{s^2-k^2}$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$e^{at}f(t)$	$F(s\text{-}a)$
$\int_0^t f(u)g(t-u)du$	$F(s)G(s)$
$u(t)$	$\frac{1}{s}$
$u(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$\delta(t)$	1
$\delta(t-a)$	e^{-as}

Table of Z-Transforms

$f(n)$	$F(z)$
$u(n)=1$	$\frac{z}{z-1}$
a^n	$\frac{z}{z-a}$
n	$\frac{z}{(z-1)^2}$
n^2	$\frac{z(z+1)}{(z-1)^3}$
e^{bn}	$\frac{z}{z-e^b}$
$a^n f(n)$	$F\left(\frac{z}{a}\right)$
$nf(n)$	$-zF(z)$
$f(n+1)$	$zF(z)-zf(0)$
$f(n+2)$	$z^2F(z)-z^2f(0)-zf(1)$

Fourier Formulae

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{2n\pi t}{T}\right)+b_n\sin\left(\frac{2n\pi t}{T}\right)\right]$$

$$a_0=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt\ ; \quad a_n=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos\left(\frac{2n\pi t}{T}\right)dt\ ; \quad b_n=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin\left(\frac{2n\pi t}{T}\right)dt$$

A	**0**	π	2π
sin A	0	0	0
cos A	1	-1	1

Standard Integrals

$y = f(x)$	$\int f(x)\,dx$
x^n $(n \neq -1)$	$\frac{x^{n+1}}{n+1}$
e^{ax}	$\frac{1}{a}e^{ax}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\cos(ax+b)$	$\frac{1}{a}\sin(ax+b)$
$\sin(ax+b)$	$-\frac{1}{a}\cos(ax+b)$

$$\int x\sin ax\,dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}; \quad \int x\cos ax\,dx = \frac{x\sin ax}{a} + \frac{\cos ax}{a^2}$$

Trigonometric Identities

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$\sin^2 A = \frac{1}{2}(1-\cos 2A); \quad \cos^2 A = \frac{1}{2}(1+\cos 2A)$$

$$\cos(-A) = \cos A; \quad \sin(-A) = -\sin A$$