CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/2018

Module Title: Engineering Mathematics 211

Module Code:	**MATH7006**
School:	School of Science and Informatics
Programme Title:	Bachelor of Engineering (Honours) in Structural Engineering Bachelor of Engineering (Honours) in Biomedical Engineering Bachelor of Engineering (Honours) in Mechanical Engineering Bachelor of Engineering (Honours) in Chemical & Process Engineering
Programme Code:	CSTRU_8_Y2 EBIOM_8_Y2 EMECH_8_Y2 ECPEN_8_Y2
External Examiner(s):	**Dr Ann O'Shea**
Internal Examiner(s):	**Dr Maryna Lishchynska**
Instructions:	Answer **ALL** questions. Show all calculations in full.
Duration:	2 hours
Sitting:	Autumn 2018
Requirements for this examination:	Mathematical Tables

Note to Candidates: Please check the **Programme Title** and the **Module Title** to ensure that you have received the correct examination.

If in doubt please contact an Invigilator.

Q1.

a) Solve the following initial value problem. Give your answer in the explicit form.

$$x\frac{dy}{dx} + 2y = 4x^2 \qquad y(1) = -2$$ **[8 marks]**

b) A metal bar with initial temperature of 25°C is dropped into a container of boiling water. Newton's law of cooling/heating is given by $\frac{d\theta}{dt} = k(\theta - \theta_m)$ where $\theta(t)$ is the temperature (in °C) at time t (in seconds), θ_m is the ambient temperature and constant $k = -0.5$.

(i) Use the above given facts to form the corresponding differential equation and write the initial condition mathematically.

(ii) Use the variables separation method to solve this differential equation and find the temperature of the bar as a function of time, $\theta(t)$.

(iii) Find the time required for the bar to reach 92°C.

[10 marks]

c) In answering parts (i) and (ii) you are required to use the method of undetermined coefficients. No marks will be awarded if any other method is used.

(i) Find the general solution of the following differential equation

$$y'' - 4y' + 4y = 12e^{2x}$$ **[10 marks]**

(ii) Find the particular solution of the following differential equation

$$\frac{d^2x}{dt^2} + 9x = 18 \qquad x(0) = 3;\ x'(0) = 6$$ **[9 marks]**

P.T.O.

d) An electrical circuit containing an inductor, a resistor and a capacitor is modelled by the following differential equation:

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

where $i(t)$ is the current at time t.

Assume the inductance $L = 1$ henry, resistance $R = 6$ ohms and the capacitance $C = 0.125$ farads. Initially, the current is zero and $\frac{di}{dt} = 10$. Use the method of undetermined coefficients to solve this initial value problem and find the current $i(t)$.

[8 marks]

P.T.O.

Q2.

a) Find the inverse Laplace transform of the following expression:

$$\frac{16}{s^2\left(s^2+4\right)}$$

[7 marks]

b) Use Laplace transforms to solve the following differential equations:

(i) $$\frac{dy}{dt}+2y=8e^{-3t} \qquad y(0)=4$$

[7 marks]

(ii) $$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=6e^{-t} \qquad x(0)=x'(0)=0$$

[8 marks]

c) A system has the transfer function:

$$T.F.=\frac{s}{(4s+1)\left(s^2+4s+5\right)}.$$

(i) Find all poles and zeros of the transfer function and sketch the pole-zero diagram.

(ii) State whether the physical system is stable or not and justify your answer.

[8 marks]

P.T.O.

Q3.

(a) V is the prism with vertices (0,0,0), (1,0,0), (1,2,0), (0,0,3),(1,0,3) and (1,2,3).

(i) Given that C is the perimeter of the base of this prism, sketch the path C and evaluate the line integral

$$\oint_C 12y^3\,dx + 6xy\,dy$$

[10 marks]

(ii) Also evaluate the triple integral

$$\iiint_V 6yz^2\,dV$$

[6 marks]

(b) A semi-circular lamina is modelled by region $R: x^2 + y^2 \leq 4$ and $y \geq 0$. Given that the density $\rho = 8$ (in kg/m^2) and the unit of length is meter determine

(i) the mass of the lamina; **[3 marks]**

(ii) the centre of mass of the lamina. **[6 marks]**

Table of Laplace Transforms

$f(t)$	$F(s)$
a	$\frac{a}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin wt$	$\frac{w}{s^2+w^2}$
$\cos wt$	$\frac{s}{s^2+w^2}$
$\sinh kt$	$\frac{k}{s^2-k^2}$
$\cosh kt$	$\frac{s}{s^2-k^2}$
$e^{at}f(t)$	$F(s\text{-}a)$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\int_0^t f(u)\,du$	$\frac{1}{s}F(s)$

$$\cosh t = \frac{e^t + e^{-t}}{2}; \quad \sinh t = \frac{e^t - e^{-t}}{2}$$

Useful Formulae

$$\bar{x} = \tfrac{1}{m}\iint_R x\,\rho(x,y)dA, \qquad \bar{y} = \tfrac{1}{m}\iint_R y\,\rho(x,y)dA$$

$$I_x = \iint_R y^2\,\rho(x,y)dA, \qquad I_y = \iint_R x^2\,\rho(x,y)dA$$