CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2018

Module Title: Mathematics for Science 3.1

Module Code: **MATH 7010**

School: School of Science and Informatics: Physical Sciences

Programme Title:

Bachelor of Science in Applied Physics & Instrumentation – Award

Programme Code:

SPHYS_7_Y3

External Examiner: **Dr. A. O'Shea**
Internal Examiner: **Ms. M. Brennan**

Instructions: Answer all FOUR questions (worth 25 marks each). <u>Table 1</u> for Q4 is attached to the exam paper. If you complete the table please insert it into your exam booklet with your name clearly written on it.

Duration: **2 HOURS**
Sitting: Autumn 2018
Requirements for this examination: Graph paper, Maths Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

1. (a) Write $f(t)$ in terms of unit step functions.

$$f(t) = \begin{cases} 5 & \text{for } 0 \leq \ t < 1 \\ -5t + 10 & \text{for } 1 \leq \ t < 2 \\ 0 & \text{for } t \geq 2 \end{cases}$$

(7 Marks)

(b) Use the Second Translation Theorem to determine Laplace transform of $(5t - 6)\mathcal{U}(t - 3)\}$.

(5 Marks)

(c) Find the inverse Laplace transform of

(i) $F(s) = \dfrac{4s^2 + 13s - 17}{s^2 + 3s - 4}$ (7 Marks)

(ii) $F(s) = \dfrac{3s + 5}{s^2 + 2s - 8}e^{-4s}$ (6 Marks)

2. (a) The response of a given system to a dirac delta function $f(t) = \dfrac{1}{6}\delta(t - \pi)$ is given by $y(t) = 5 - 1.4e^{-7t}$.

(i) Find the transfer function. (5 Marks)

(ii) Find the poles of the transfer function. (4 Marks)

(iii) Sketch the poles on the s plane. (2 Marks)

(iv) Sketch the transient responses of the poles. (2 Marks)

(v) Describe the stability of the system. (2 Marks)

(b) Solve the differential equation

$$2.4y'(t) + y(t) = 4.8\mathcal{U}(t - 3)$$

subject to the initial condition $y(0) = 0$. Sketch the solution. (10 Marks)

3. (i) Sketch the graph of the squarewave periodic function $f(t)$ in the interval $[-2\pi, 2\pi]$.

$$f(t) = \begin{cases} 0 & \text{for } -\pi < t < 0 \\ 4 & \text{for } \quad 0 < t < \pi \end{cases}, \qquad f(t+2\pi) = f(t)$$

(3 Marks)

(ii) Find the Fourier series representation of $f(t)$. (14 Marks)

(iii) Use your result from (ii) to write down the Fourier series representation of the function

$$g(t) = \begin{cases} 0 & \text{for } -5 < t < 0 \\ 4 & \text{for } \quad 0 < t < 5 \end{cases}, \qquad g(t+10) = g(t)$$

(4 Marks)

(iv) Use your result from (ii) to write down the Fourier series representation of the function

$$h(t) = \begin{cases} 0 & \text{for } -\pi < t < 0 \\ 6 & \text{for } \quad 0 < t < \pi \end{cases}, \qquad h(t+2\pi) = h(t)$$

(4 Marks)

4. (a) A fuel company believes that there is a linear relationship between the average cost of diesel (to the nearest cent per litre), y, and the month of the year, x. The following data have been collected for the first six months of a particular year.

x	1	2	3	4	5	6
y	122	124	125	126	128	127

(i) Draw a scatter plot for the given data. (3 Marks)

(ii) Use the Least Squares Method to determine the regression line equation. Use **Table 1** on page 6. (14 Marks)

(iii) Plot the regression line on the scatter plot. (2 Marks)

(iv) Predict the average cost per litre of petrol when $x = 4.5$. (1 Mark)

(b) Consider the following waveform.

$$f(t) = \sin(\pi t) + 0.7\sin(4\pi t) - 0.4\cos(6\pi t)$$

(i) State the fundamental angular frequency of $f(t)$. (1 Mark)

(ii) Describe the frequency and amplitude characteristics of the second and the sixth harmonics of $f(t)$. (4 Marks)

Useful formula:

- General Lagrange Interpolation Polynomial: for n a positive integer

$$\mathbf{f(x) \approx p_n(x) = \sum_{k=0}^{n} L_k(x) f_k = \sum_{k=0}^{n} \frac{l_k(x)}{l_k(x_k)} f_k}$$

where $L_k(x_k) = 1$ and 0 at the other nodes.

- Normal Equations:

$$\Sigma y = m\Sigma x + Nc$$

$$\Sigma xy = m\Sigma x^2 + c\Sigma x$$

where N is the number of ordinates.

- Newton-Gregory Interpolation (forward difference) Formula

$$\mathbf{y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \ldots}$$

- Representation of a periodic function by a trigonometric Fourier series:

If the function $f(t)$ has period $T = 2L$ then

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi}{L}t + b_n \sin\frac{n\pi}{L}t)$$

with the Fourier coefficients of $f(t)$ given by the Euler formulas

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(t)dt$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi}{L}tdt \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\frac{n\pi}{L}tdt. \quad n = 1, 2, 3, \ldots$$

MATH7010: Table of Laplace Transforms

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$ $\qquad$ $\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}F(s)$

$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$ $\qquad$ $\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$

$\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}$ $\qquad$ $\mathcal{L}\{\mathcal{U}(t-a)\} = \dfrac{e^{-as}}{s}$

$\mathbf{f(t)}$	$\mathbf{F(s)}$
1	$\dfrac{1}{s}$
t^n	$\dfrac{n!}{s^{n+1}}$
$e^{-\alpha t}$	$\dfrac{1}{s+\alpha}$
$\sin \omega t$	$\dfrac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\dfrac{s}{s^2+\omega^2}$
$te^{-\alpha t}$	$\dfrac{1}{(s+\alpha)^2}$
$e^{-\alpha t} - e^{-\beta t}$	$\dfrac{\beta-\alpha}{(s+\alpha)(s+\beta)}$
$e^{-\alpha t}\sin \omega t$	$\dfrac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}\cos \omega t$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Name: ______________________________

Q4(ii)

Table 1

x	y		
1	122		
2	124		
3	125		
4	126		
5	128		
6	127		