**CORK INSTITUTE OF TECHNOLOGY**

**INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ**

**Autumn Examinations 2017/2018**

# Module Title: Maths for Digital Systems

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| **Module Code:** | MATH7011 |
| **School:** | Mechanical, Electrical and Process Engineering |
| **Programme Title:** | Bachelor of Engineering (Honours) in Electronic Engineering – Year 3<br>Bachelor of Engineering in Electronic Engineering – Year 3 |
| **Programme Code:** | CR_EELES_8 CR_EELXE_7 |
| **External Examiners(s):** | Dr. A. O'Shea |
| **Internal Examiners(s):** | Dr. M. Nicholson |
| **Instructions:** | Answer **all** questions.<br>Questions 1 and 2 carry 33 marks each. Question 3 carries 34 marks.<br>You must **show your work** or explain your solution, otherwise points may be deducted.<br>No credit will be given for incorrect steps nor will credit be given for correct solutions arrived at by incorrect means. |
| **Duration:** | 2 Hours |
| **Sitting:** | Autumn 2018 |
| **Requirements for this examination:** | Mathematical Tables |

**Note to Candidates:**

Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

## Question 1.

(a) Consider $R$, a binary reprtition code of length 3.

(i) List all the codewords of $R$.

(ii) Suppose that a codeword is sent, and that two bits are corrupted. Will the errors be detected? Will the correct decoding decision be made? Justify your answers.

(4 marks)

(b) A binary symmetric channel has cross-over probability $p = 0.15$. Find the probability that a word of length 6 is corrupted in exactly two places during transmission. (4 marks)

(c) (i) Complete the following decoding table for a binary linear $[5,3]$ code $B$:

| 00000 | 10010 | 01011 | 01100 | | 11001 | 11110 | 10101 |
|---|---|---|---|---|---|---|---|
| 10000 | | | | | | | |
| 01000 | | | | 01111 | | | |
| | | | | | | | |

(ii) Suppose that the codeword 11001 is sent, and that the word 11011 is received. Will the correct decoding decision be made? Explain why/why not.

(iii) Find in terms of the crossover probability $p$ a formula for $P_{undetect}(B)$, the probability that the code $B$ fails to detect an error.

(11 marks)

(d) (i) Write down a parity check matrix $H$ for a Hamming $[3,2]$ code.

(ii) Hence encode the message 1110, identifying the redundancy bits.

(iii) Decode the received word y = 1100000.

(14 marks)

**Question 2.**

(a) (i) Find the $\mathcal{Z}$−transform of the sequence $f[n] = 3(-1)^n + 5\delta[n]$.

(ii) Find the first four terms of $f[n+1]$ and its $\mathcal{Z}$−transform.

(7 marks)

(b) Let $x[n]$ be the sequence obtained when the function $g(t) = e^{-5t}$ is sampled at $t = 0$ and at intervals of 0.002 seconds thereafter.

(i) Determine the first four values of the sequence $x[n]$.

(ii) Find a formula for $x[n]$ in terms of $n \in \{0, 1, 2, 3, \ldots\}$.

(iii) Hence find $X(z)$, the $\mathcal{Z}$-transform of the sequence $x[n]$.

(6 marks)

(c) Consider the following first order difference equation:

$$x[n+1] - 2x[n] = (-1)^n; \qquad x[0] = 1$$

(i) Find $x[1]$ and $x[2]$ by using the recursion method.

(ii) Solve this difference equation using $\mathcal{Z}$-transforms.
The following information is useful:

$$\frac{1}{(z+1)(z-2)} = \frac{1}{3}\left[\frac{1}{z-2} - \frac{1}{z+1}\right]$$

(11 marks)

(d) A system has $z$-transfer function

$$T(z) = \frac{z}{(2z+1)(z^2+2z+5)}$$

(i) Find all the poles and zeros of this system.

(ii) Sketch a pole–zero diagram.

(iii) Is the system $T(z)$ stable? Justify your answer.

(9 marks)

**Question 3.**

(a) Let

$$x(t) = 5\cos(t) + 12\sin(t).$$

(i) Write the complex fourier series expansion of $x(t)$.

(ii) Find the fourier coefficents of $x(t)$.

(iii) Plot the frequency spectrum of $x(t)$.

(10 marks)

(b) Compute the Fourier Transform $X(\omega)$ of

$$x(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

where $u(t)$ is the unit step function. (4 marks)

(c) Let $f[n]$ is the following 4-point sequence:

$$f[0] = 3, \qquad f[1] = 4, \qquad f[2] = 4, \qquad f[3] = 5.$$

(i) Find $F[k] = \mathcal{D}\{f[n]\}$.

(ii) Sketch the phase spectrum $\phi(k)$.

(iii) Sketch the magnitude spectrum $|F(k)|$.

(14 marks)

(d) Let $\omega_8$ denote the value of $\omega = e^{-2j\pi/N}$ for an 8-point DFT.
Calculate the value of each of the following 'twiddle factors':

$$\omega_8, \qquad \omega_8^2, \qquad \omega_8^3$$

(6 marks)

Cork Institute of Technology

# MATH7011

# Mathematics for Digital Systems

## Tables and Formulae Autumn 2018

This booklet contains:

- Table of $\mathcal{Z}$-transforms;
- Table of $\mathcal{F}$-transforms;
- Formulae of Complex Fourier Series;
- Euler's Formulae;
- Formulae for Discrete Fourier Analysis;

Candidates may also request a copy of the State Examinations Commission's *formulae and tables* booklet.

## Table of $\mathcal{Z}$— Transforms

| $f(n)$ | $F(z)=\mathcal{Z}\{f(n)\}=\sum_{n=0}^{\infty} f(n)z^{-n}=\sum_{n=0}^{\infty}\frac{f(n)}{z^n}$ |
|---|---|
| $\delta(n)$ | $1$ |
| $a^n$ or $a^n u(n)$ | $\frac{z}{z-a}$ |
| $u(n)$ or $1$ or $1^n$ | $\frac{z}{z-1}$ |
| $na^{n-1}$ | $\frac{z}{(z-a)^2}$ |
| $n$ | $\frac{z}{(z-1)^2}$ |
| $n(n-1)$ | $\frac{2z}{(z-1)^3}$ |
| $n^2$ | $\frac{z^2+z}{(z-1)^3}$ |
| $\cos \omega n$ | $\frac{z(z-\cos\omega)}{z^2-2z\cos\omega+1}$ |
| $\sin \omega n$ | $\frac{z\sin\omega}{z^2-2z\cos\omega+1}$ |

**Table of $\mathcal{Z}$— Transforms Continued**

| $f(n)$ | $F(z)$ |
|---|---|
| $f(n+1)$ | $zF(z)-zf(0)$ |
| $f(n+2)$ | $z^2F(z)-z^2f(0)-zf(1)$ |
| $f(n-m)u(n-m)$ | $z^{-m}F(z)$ |

## Table of $\mathcal{F}$— Transforms

| $f(t)$ | $F(\omega) = \mathcal{F}\{f(t)\} = \sum_{t=-\infty}^{\infty} f(t)e^{-j\omega t}$ |
|---|---|
| $\delta(t)$ | $1$ |
| $u(t)$ | $\frac{1}{j\omega} + \pi\delta(\omega)$ |
| $f(t-t_0)$ | $F(\omega)e^{-j\omega t_0}$ |
| $f(\alpha t)$ | $\frac{1}{\lvert\alpha\rvert}F(\frac{\omega}{\alpha})$ |
| $e^{j\omega_0 t}$ | $2\pi\delta(\omega-\omega_0)$ |
| $f(t)e^{j\omega_0 t}$ | $F(\omega-\omega_0)$ |
| $\cos(\omega_0 t)$ | $\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$ |
| $\sin(\omega_0 t)$ | $\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$ |
| $e^{-a\lvert t\rvert}$ | $\frac{2a}{a^2+\omega^2}$ |
| $u(t)e^{-at}$ | $\frac{1}{a+j\omega}$ |
| $u(t)te^{-at}$ | $\frac{1}{(a+j\omega)^2}$ |

## Formulae — Complex Fourier Series

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}, \text{ where } a_k = \frac{1}{T}\int_0^T f(t)e^{-jk\omega_0 t}\,dt$$

## Euler's Formulae

$$e^{jt} = \cos(t) + j\sin(t)$$

$$\cos(t) = \frac{e^{jt} + e^{-jt}}{2}$$

$$\sin(t) = \frac{e^{jt} - e^{-jt}}{2j} = \frac{je^{-jt} - je^{jt}}{2}$$

## Formulae — Discrete Fourier Analysis

**Discrete Fourier Transform**

$$\mathcal{D}\{f(n)\} = F(k) = \sum_{n=0}^{N-1} f(n)e^{-j\frac{2\pi}{N}nk} \qquad \text{for } k = 0, 1, 2, \ldots, N-1$$

Matrix representation

$$\begin{bmatrix} \omega^0 & \omega^0 & \omega^0 & \omega^0 & \ldots & \omega^0 \\ \omega^0 & \omega & \omega^2 & \omega^3 & \ldots & \omega^{N-1} \\ \omega^0 & \omega^2 & \omega^4 & \omega^6 & \ldots & \omega^{2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \ldots & \vdots \\ \omega^0 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{3(N-1)} & \ldots & \omega^{(N-1)^2} \end{bmatrix} \qquad \omega = e^{-j\frac{2\pi}{N}}$$

**Inverse Transform**

$$\mathcal{D}^{-1}\{F(k)\} = f(n) = \frac{1}{N}\sum_{k=0}^{N-1} F(k)e^{j\frac{2\pi}{N}nk} \qquad \text{for } n = 0, 1, 2, \ldots, N-1$$

Matrix representation

$$\frac{1}{N}\begin{bmatrix} \omega^0 & \omega^0 & \omega^0 & \omega^0 & \ldots & \omega^0 \\ \omega^0 & \omega^{N-1} & \omega^{2(N-1)} & \omega^{3(N-1)} & \ldots & \omega^{(N-1)^2} \\ \vdots & \vdots & \vdots & \vdots & \ldots & \vdots \\ \omega^0 & \omega^2 & \omega^4 & \omega^6 & \ldots & \omega^{2(N-1)} \\ \omega^0 & \omega & \omega^2 & \omega^3 & \ldots & \omega^{N-1} \end{bmatrix} \qquad \omega = e^{-j\frac{2\pi}{N}}$$