CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 1 Examinations 2017/18

Module Title: Maths for Electronic Engineering

Module Code: **MATH 7013**

School: School of Electrical and Electronic Engineering

Programme Title:

Bachelor of Engineering in Electronic Engineering – Year 3

Programme Code: EELXE_7_Y3
EELES_7_Y3

External Examiner(s): **Dr. A. O'Shea**
Internal Examiner(s): **Dr. V. Morari**

Instructions: **Answer ALL THREE questions**

Duration: **2 HOURS**

Sitting: Autumn 2018

Requirements for this examination: Mathematics Tables

The accompanying booklet contains:

Table of Laplace transforms
Formulae for Fourier Analysis

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.
If in doubt please contact an Invigilator.

1. (a) Find $\mathcal{L}\left\{\frac{t^4}{4} + 3t^2 + 2t + \pi\right\}$.

[4 marks]

(b) Find $\mathcal{L}\{3e^{6t}\sin t\}$. Show all workings.

[6 marks]

(c) Find $\mathcal{L}^{-1}\left\{\frac{1}{s-4} + \frac{6}{2s+2} + \frac{2}{s^8}\right\}$.

[6 marks]

(d) Find $\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2(s-1)}\right\}$.

[8 marks]

(e) Using the result above, or otherwise, solve the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x(t) = e^t, \;\; x(0) = 0, \;\; x'(0) = 0.$$

[10 marks]

[P.T.O]

2. (a) Plot a pole-zero map for the system

$$G(s) = \frac{s}{(s-4)(s^2+4s+5)}.$$

(i) Comment on the stability of this system.

(ii) Which, if any, of the poles of this system give rise to oscillations?

[6 marks]

(b) Consider the second order differential equation

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 8x(t) = r(t) \text{ given that } x(0) = x'(0) = 0.$$

(i) Find the transfer function for the input-output system governed by this differential equation.

[4 marks]

(ii) Find the total response to the input $r(t) = 2e^{3t}$, indicating the transient response and the steady state response. [A complete solution is required here.]

[9 marks]

(c) Find the Laplace Transform of the function

$$k(t) = \cos(t-2)u(t-2)$$

[4 marks]

(d) Find the inverse Laplace transform $g(t)$ for the function

$$G(s) = \frac{2e^{-3s} - 4e^{-s}}{s}.$$

and plot the function $g(t)$

[10 marks]

[P.T.O]

3. Consider the function defined by

$$f(t) \;=\; \begin{cases} 0 \;; & -0.1 \le t < 0 \\ 3 \;; & 0 \le t < 0.1 \end{cases}$$

and $f(t + 0.2) = f(t)$ for all $t \in \mathbb{R}$.

(a) Draw a graph of the function $f(t)$ from $t = -0.3$ to $t = 0.3$.

[4 marks]

(b) Find the Fourier series representation of the function $f(t)$.

[17 marks]

(c) Sketch the Fourier amplitude spectrum of this waveform.

[7 marks]

(d) Find the power content P_{av} of the function $f(t)$.

[5 marks]

[P.T.O]

Some Properties of the Laplace Transform

Definition and Notation

$$\mathcal{L}\{f(t)\} \equiv F(s) = \int_0^\infty e^{-st} f(t) dt$$

Linearity:

For any constants c_1 and c_2

$$\mathcal{L}\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 \mathcal{L}\{f_1(t)\} + c_2 \mathcal{L}\{f_2(t)\}$$

The First Shift Theorem:

$$\mathcal{L}\left\{e^{-at} f(t)\right\} = F(s+a)$$

The Second Shift Theorem:

$$\mathcal{L}\{f(t-c)u(t-c)\} = e^{-cs} F(s)$$

Derivative:

$$\mathcal{L}\left\{\frac{df}{dt}\right\} = \mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\left\{\frac{d^2 f}{dt^2}\right\} = \mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

Short Table of Laplace Transforms

$f(t)$	$F(s) = \int_0^\infty e^{-st} f(t)dt$
k	$\frac{k}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$\sin bt$	$\frac{b}{s^2+b^2}$

Fourier Series Representation

If $f(t)$ is periodic with period T then

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2\pi nt}{T}\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2\pi nt}{T}\,dt$$

Fourier Series of the function f is then given by:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n\cos\frac{2\pi nt}{T} + b_n\sin\frac{2\pi nt}{T}\right\}$$

Average Power Content

If the function $f(t)$ is periodic with period T then the power content, P_{av}, of $f(t)$ is defined via

$$P_{av} = \frac{1}{T}\int_0^T [f(t)]^2\,dt.$$

It is the mean of the square of $f(t)$.

Parseval's Theorem

If the function $f(t)$ is periodic with period T and has Fourier coefficients a_n and b_n so that

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n\cos\frac{2\pi nt}{T} + b_n\sin\frac{2\pi nt}{T}\right\}$$

then

$$P_{av} = \frac{1}{T}\int_0^T [f(t)]^2\,dt = \left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\frac{a_n^2+b_n^2}{2}.$$

Amplitude Phase Representation

The amplitude-phase form of the Fourier representation of $f(t)$ is given by

$$f(t) = A_0 + \sum_{n=1}^{\infty}\Big[A_n\cos(nt-\phi_n)\Big]$$

where $A_0 = \frac{a_0}{2}$, $A_n = \sqrt{a_n^2+b_n^2}$ and $\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

Discrete Frequency Spectra

A plot of

$$A_n \text{ versus } n$$

for each $n \in N$ is an amplitude spectrum of $f(t)$.

A plot of

$$\phi_n \text{ versus } n$$

for each $n \in N$ is a phase spectrum of $f(t)$.

Between them, these spectra constitute the discrete frequency spectra.