CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/18

Module Title: Numerical Methods 2

Module Code:	**MATH7016**
School:	School of Mechanical and Process Engineering
Programme Title:	Bachelor of Engineering (Hons) in Mechanical Eng. – Y. 2
Programme Code:	EMECH_8_Y2
External Examiner(s):	Dr Ann O'Shea
Internal Examiner(s):	**Dr J.P. McCarthy**
Instructions:	**Section A: Answer ALL Questions** **Section B: Answer Question B1 OR Question B2**
Duration:	2 Hours
Sitting:	Autumn 2018
Exam Requirements:	Computer, Answer Booklet.
NB:	Please find helpful formulae and tables at the back of this exam paper. You will submit both an answer booklet (Section A) and a (macro-enabled) Excel file (Section B)

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.

If in doubt about instructions please ask the invigilator for help.

SECTION A: Written [60 Marks]

Answer all questions in Section A.

A1: Consider the initial value problem

$$\frac{dy}{dx} = x + 2y(x)^2\,;\ \ y(1) = 0.$$

Use Heun's Method with a step-size of 0.1 to approximate $y(1.3)$. Make calculations correct to four or more significant figures.

[15 Marks]

A2: Consider an initial value problem

$$\frac{dy}{dx} = F(x,y)\,;\ \ y(x_0) = y_0.$$

Using k-th order Runge-Kutta with a step-size of h, it can be shown that the *local error* is $\mathcal{O}(h^{k+1})$.

(a) What does it mean to say that the local error is $\mathcal{O}(h^{k+1})$?

[2 Marks]

(b) Show that if we use a k-th order Runge-Kutta to approximate $y(x_n) = y(x_0 + n \cdot h)$, that the *global error* is $\mathcal{O}(h^k)$.

[6 Marks]

(c) Why is k-th order Runge-Kutta easier to use than the $(k+1)$-Term-Taylor Method?

[2 Marks]

(d) If $k = 4$, what is the effect on the local error if we half the step-size?

[2 Marks]

(e) If $k = 3$, what is the effect on the global error if we quarter the step-size?

[2 Marks]

A3: What is the difference between an Initial Value Problem and Boundary Value Problem?
[4 Marks]

A4: Suppose the temperature at a point a distance x along a rod of length 6 m is given by the linear ode:

$$\frac{d^2T}{dx^2} + 0.03(40 - T) = 0\,; \;\; T(0) = 0 \;\&\; T(4) = 20.$$

Using a step-size of $h = 1$, use the (Euler) Shooting Point Method to approximate $T(x)$ for $x = 1$ & 2 m.

Make calculations correct to four or more significant figures.
[20 Marks]

A5: Use Finite Differences to show that, approximately, Laplace's Equation (on a thin plate)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0,$$

implies the 'discrete' Mean Value Property on a square grid:

$$T = \text{mean(temperature at adjacent nodes)}.$$

[7 Marks]

SECTION B: VBA Programming [40 Marks]

Answer Question B1 or Question B2.

B1: The charge at time t; $q = q(t)$, on an initially uncharged capacitor, of capacitance C, in series with a resistor of resistance R and connected to a battery of voltage E, is given by the solution of the initial value problem:

$$\frac{dq}{dt} = \frac{E}{R} - \frac{q(t)}{RC}\,; \qquad q(0) = 0. \tag{1}$$

Write a VBA program that takes as input

- a step-size h
- a final t value t_f
- the capacitance C, resistance R, and voltage E,

and implements Euler's Method to produce a table of (t and) T values from $t = 0$ to at least t_f.

[40 Marks]

B2: The transient-state temperature of an insulated rod of length 8, subject to boundary conditions, is given by:

$$0.8 \cdot \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}.$$

Where $T(x,t)$ is the temperature at a distance x along the rod at time t, the initial and boundary conditions are given by:

$$T(0,0) = 80, \qquad T(10,0) = 50, \qquad T(x,0) = 0 \text{ (for } 0 < x < 10)$$
$$T(0,t) = 100, \qquad T(10,t) = 50,$$

i.e. the rod is initially at a temperature of 0 while the temperatures on the endpoints are fixed at 100 and 50.

Write a VBA program that implements the Finite Difference Method to approximate the transient temperature at three internal points of the rod at time intervals of 0.2. The program should run until the difference between the transient-state temperatures and the steady-state temperature is less than 0.5.

[40 Marks]

Useful Code

```
cells.clear
```

Variables and their Data Types

```
 Dim dblX As Double
 Dim intC As Integer
```

Do Loops

You need starting values and counters — which also need starting values.

```
Do While/Until CONDITION
STUFF
Loop

Do
STUFF
Loop While/Until CONDITION
```

$$y_{i+1} = y_i + h \cdot F(x_i, y_i)$$

$$\begin{aligned} y^0_{i+1} &= y_i + h \cdot F(x_i, y_i) \\ y_{i+1} &= y_i + h \cdot \frac{F(x_i, y_i) + F(x_{i+1}, y^0_{i+1})}{2} \end{aligned}$$

$$v(0) = v_a + \frac{y(x_1) - y_a}{y_b - y_a}(v_b - v_a).$$

$$\left.\frac{dy}{dx}\right|_{x_i} \approx \frac{y(x_{i+1}) - y(x_i)}{\Delta x}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x_i} \approx \frac{y(x_{i+1}) - 2y(x_i) + y(x_{i-1})}{(\Delta x)^2}$$