CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2018

Module Title: Technological Mathematics 301

Module Code: **MATH 7020**

School: School of Mechanical, Electrical and Process Engineering:
Mechanical, Biomedical & Manufacturing Engineering
Process, Energy and Transport Engineering

Programme Title:

Bachelor of Engineering in Biomedical Engineering – Award
Bachelor of Engineering in Mechanical Engineering – Award
Bachelor of Engineering in Sustainable Energy Engineering (Hons) – Year 3

Programme Code:

EBIME_7_Y3 EMECH_7_Y3 ESENT_8_Y3

External Examiner: **Dr. A. O'Shea**
Internal Examiner(s): **Ms. M. Brennan**

Instructions: Answer all FOUR questions (worth 25 marks each)

Duration: **2 HOURS**
Sitting: Autumn 2018
Requirements for this examination: Graph paper, Maths Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

1. (a) Find the particular solution of the first order separable differential equation given that y(0)=2.

$$(4x^9 + 6)dy = 12x^8 y dx$$

(7 marks)

(b) Show the integrating factor of $2\frac{dy}{dx} = -18x^2y + x^2$ is e^{3x^3}. Hence find the general solution of the differential equation.

(8 marks)

(c) Use the Method of Undetermined Coefficients to find the general solution of the non-homogenous second order differential equation $y''(x) - 6y'(x) + 8y(x) = x^2 + 3x$.

(10 marks)

2. (a) Use Euler's Method with $h = 0.2$ to obtain an approximation to $y(2.7)$ given that y satisfies the differential equation $y' + 0.6\sqrt{y+5} = 2x^2$, and $y(2.3) = 4$.

(10 marks)

(b) A tank contains liquid and the tank is in the form of an inverted circular cone of height $5m$ and base diameter 0.5 m. Let $h(t)$ be the height of liquid present in the tank at any instant t (seconds). The tank is emptied through a circular orifice at the base of diameter 0.01 m and due to viscosity and surface tension the velocity of the emerging liquid is given by $0.6\sqrt{2gh}$ where $g = 9.81ms^{-2}$.

(i) Show that the volume V of liquid is given by $V = \frac{\pi h^3}{1200}$.

(3 marks)

(ii) Show the differential equation representing the height $h(t)$ of liquid at time t (seconds) is

$$h^2 \frac{dh}{dt} = -2.658 \times 10^{-2}\sqrt{h}$$

(6 marks)

(iii) Solve this differential equation if initially the tank is full.

(3 marks)

(iv) Determine how long it takes (in hours) to empty the full tank.

(3 marks)

3. (a) Find the Laplace transform of

(i) $f(t) = 8t^5 + e^{-0.3t} - 9te^{10t}$ (ii) $f(t) = 3\cos^2(6t)$ (iii) $f(t) = \frac{1}{7}t^4e^{-0.8t}$

(10 marks)

(b) The temperature $x(t)$ of a body at time t (seconds) is described by the differential equation

$$14x'(t) + x(t) = 20$$

(i) Use Laplace transforms to solve the equation subject to the initial condition $x(0) = 2$.

(ii) Find the time taken to reach $16°C$. Find the rate at which the temperature is changing at this time.

(15 marks)

4. (a) Find the inverse Laplace transform of

(i) $F(s) = \dfrac{12}{(s+8)(2s-4)}$

(5 marks)

(ii) $F(s) = \dfrac{s+11}{(s-9)^2+16}$

(5 marks)

(b) The differential equation governing the displacement $x(t)$ of a damped oscillator is given by

$$x''(t) + 2x'(t) + 10x(t) = 0$$

(i) Use Laplace transforms to solve the differential equation subject to $x(0) = 0$ and $x'(0) = 6$.

(9 marks)

(ii) Determine the period of the oscillation, the duration of the oscillations and the number of oscillations.

(3 marks)

(iii) Sketch the solution $x(t)$, clearly indicating the period and the duration of the oscillations on the sketch.

(3 marks)

Short Table of Laplace Transforms

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

f(t)	**F(s)**
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$te^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$
$e^{-\alpha t} - e^{-\beta t}$	$\frac{\beta-\alpha}{(s+\alpha)(s+\beta)}$
$e^{-\alpha t} \sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t} \cos \omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$