CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/18

Module Title: Technological Maths 312

Module Code: **MATH7021**

School: School of Building and Civil Engineering

Programme Title: Bachelor of Engineering in Civil Engineering – Year 3
Bachelor of Engineering in Environmental Engineering – Year 3

Programme Code: CCIVL_7_Y3
CCENVI_7_Y3

External Examiner(s): Dr Ann O'Shea

Internal Examiner(s): **Dr J.P. McCarthy**

Instructions: **Answer** ALL **Questions**

Duration: 2 Hours

Sitting: Autumn 2018

Exam Requirements: Mathematics Tables.

NB: Please find helpful formulae and tables at the back of this exam paper

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.

If in doubt please contact an Invigilator.

1. (a) In trying to find the partial fraction expansion of a rational function, the following linear system had to be solved:

$$\begin{aligned} A + B + 2C + 3D &= 0 \\ 3A + 4B + 6C + 5D &= 1 \\ 3A + 5B + 7C + 8D &= -2 \\ 4A + 5B + 9C + 5D &= 1 \end{aligned}$$

Use *Gaussian Elimination* to find the solution of the simultaneous equations.

[10 Marks]

(b) i. Use **only** *Gaussian elimination* to find the intersection of the lines:

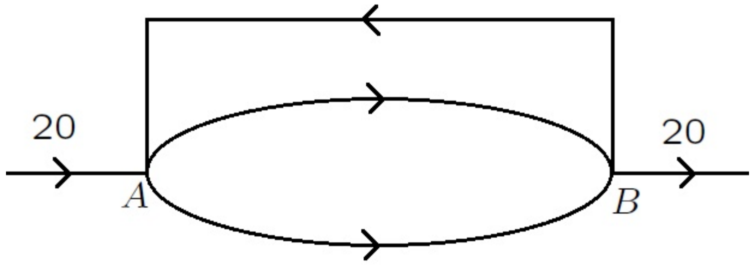
$$\begin{aligned} 6x + 9y &= 18 \\ -4x - 6y &= -13 \end{aligned}$$

[4 Marks]

ii. Explain this result geometrically.

[1 Mark]

(c) Consider the following traffic flow diagram:



i. Identify the variable/unknown flows.

[1 Mark]

ii. Write down the linear system governing the system flow.

[2 Marks]

iii. Use *Gaussian elimination* to solve the linear system.

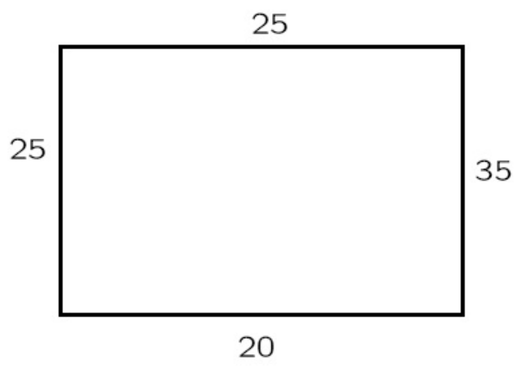
[4 Marks]

(d) Solve the following linear system subject to the constraint that all calculations are made correct to **three significant figures only.**

$$\begin{aligned} 0.004x + 0.15y &= -1.24 \\ 0.2x + 0.3y &= 1.85 \end{aligned}$$

[6 Marks]

(e) Suppose the following plate has dimensions 4:2 and is subject to boundary temperatures as shown:



i. Using an appropriate grid, find a linear system whose solution approximates the heat distribution of the plate at internal grid-points using the *Mean-Value Property*.

[3 Marks]

ii. Starting with an appropriate approximation, use two iterations of the *Jacobi Method* to approximate the solution of the linear system. Use three significant figures for all calculations.

[4 Marks]

2. (a) Find, using **only** the method of undetermined coefficients, the general solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y(x) = e^{2x}.$$

[7 Marks]

(b) Solve, using **only** the method of undetermined coefficients, the initial value problem

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x(t) = -18\,; \qquad x(0) = 4\,,\; x'(0) = 0.$$

[8 Marks]

3. (a) Suppose that the bending moment $b(x)$ at a distance x along a beam satisfies the first order differential equation:

$$\frac{d^2b}{dx^2} = -12\,; \qquad\qquad b(0) = 0\,,\ \ b'(0) = 36$$

Use **only** *Laplace Methods* to solve the differential equation for $b(x)$.

[8 Marks]

(b) The differential equation governing the displacement $x(t)$ of a *damped oscillator* is given by:

$$x''(t) + 2x'(t) + 10x(t) = 0\,; \qquad\qquad x(0) = 0\,,\ \ x'(0) = 1.$$

i. Solve the differential equation using **only** *Laplace transforms.*

[9 Marks]

ii. Suppose that the differential equation models a door closer. Is it well designed? Briefly justify your answer.

[1 Mark]

(c) The velocity, $v(t)$, of a parachutist subject to linear air-drag, t seconds after falling out of a plane, is given by the differential equation:

$$\frac{dv}{dt} = 9.81 - 0.025v(t)\,; \qquad\qquad v(0) = 0.$$

i. Solve the differential equation using **only** *Laplace transforms.*

[6 Marks]

ii. What is the behaviour of $v(t)$ for large values of t?

[1 Mark]

(d) Solve the following system of differential equations using *Laplace Transforms*:

$$\frac{dx}{dt} = -x(t) + y(t)\,; \qquad\qquad x(0) = 10$$

$$\frac{dy}{dt} = 2x(t) - 2y(t)\,; \qquad\qquad y(0) = 2$$

[10 Marks]

4. (a) A triangular region has vertices $(0,0)$, $(2,0)$ and $(0,1)$. Find the second moment of area of this region about the x-axis:

$$I_{xx} = \iint y^2 \, dA.$$

[7 Marks]

(b) A cylinder described by $x^2 + y^2 \leq 4$ and $0 \leq z \leq 3$ has density $\rho(x, y, z) = y^2 z$. Find the mass of the cylinder:

$$m = \iiint_V \rho(x, y, z) \, dV.$$

[HINT: $\displaystyle\int^{2\pi} \sin^2 \theta \, d\theta = \pi$]

[8 Marks]

Laplace Transform Table

The Laplace Transform is defined by

$$F(s) = \mathcal{L}\{f(t)\} := \int_0^\infty f(t)e^{-st}\,dt.$$

$f(t)$	$F(s)$
$A =$ constant	$\frac{A}{s}$
t^N	$\frac{N!}{s^{N+1}}$
$\frac{t^{N-1}}{(N-1)!}$	$\frac{1}{s^N}$
e^{-at}	$\frac{1}{s+a}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$f(t)e^{-at}$	$F(s+a)$
$f'(t)$	$s \cdot F(s) - f(0)$
$f''(t)$	$s^2 \cdot F(s) - s \cdot f(0) - f'(0)$