CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/18

Module Title: Technological Mathematics 320

Module Code: **MATH 7022**

School: School of Electrical and Electronic Engineering

Programme Title:

Bachelor of Engineering in Electrical Engineering – Award

Programme Code: EELEC_7_Y3
EEPSY_8_Y3

External Examiner(s): **Dr. A. O'Shea**

Internal Examiner(s): **Dr. V. Morari**

Instructions: **Answer ALL questions.**

Duration: **2 HOURS**

Sitting: Autumn 2018

Requirements for this examination: Mathematics Tables

The accompanying booklet contains:

Table of Laplace Transforms
Formulae for Fourier Analysis

1. (a) Find $\mathcal{L}\left\{3t + 2e^{4t} + \dfrac{2}{e^{4t}} + 10\right\}$.

[4 marks]

(b) Find $\mathcal{L}\left\{8\sin^2 4t\right\}$.

[4 marks]

(c) Find $\mathcal{L}\{e^{10t}\sin 4t\}$. Show all workings.

[6 marks]

(d) Find $\mathcal{L}^{-1}\left\{\dfrac{s+1}{s^2+2s+3}\right\}$.

[6 marks]

(e) Solve the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x(t) = e^{-2t}, \quad x(0) = 0, x'(0) = 0.$$

[14 marks]

[P.T.O]

Autumn 2018

2. (a) Sketch 3 cycles of the waveform

$$f(t) = t, \ -2\pi \leq t \leq 2\pi$$

with $f(t + 4\pi) = f(t)$.

[5 marks]

(b) Find the Fourier series representation of the function $f(t)$.

[16 marks]

(c) Sketch the Fourier amplitude spectrum of this waveform.

[5 marks]

(d) Find the power average content P_{av} of the function $f(t)$.

[7 marks]

[P.T.O]

3. (a) What is a confidence interval used for? Explain briefly in your own words.

[4 marks]

(b) Barksdale is a manufacturer of burglar alarms in Baltimore. A quality control engineer is measuring the decibel level of a particular alarm manufactured by Barksdale and intends to take a sample of such alarms to establish a 98% confidence interval for the decibel level. If a margin of error of 0.06dB is allowed, and the standard deviation of such alarms is known to be 0.8dB, find the smallest sample size that will satisfy these requirements.

What assumptions underlie your answer?

[7 marks]

(c) Stanfield is another manufacturer of burglar alarms and in competition with Barksdale. The decibel level of these alarms is known to follow a normal distribution. A random sample of six alarms is chosen and are found to have the following decibel levels.

80.4 79.1 84.3 78.0 80.1 78.9

Establish a 95% confidence interval for the decibel levels of Stanfield alarms.

[12 marks]

(d) A contract between Cork City Council and Omar Little's Road Construction company called for an asphalt road with an average thickness of 10 inches. The City Council thought that the construction company had defrauded the city by making the road less than 10 inches thick. The city then took 150 core samples 2 inch in diameter. The mean thickness of the samples was 9.5 inches and the standard deviation was 1.7 inches. Did fraud occur? Perform a suitable hypothesis test, stating your null and aternative hypotheses, and clearly stating your conclusion.

[10 marks]

Some Properties of the Laplace Transform

Definition and Notation

$$\mathcal{L}\{f(t)\} \equiv F(s) = \int_0^\infty e^{-st} f(t) dt$$

Linearity:

For any constants c_1 and c_2

$$\mathcal{L}\{c_1 f_1(t) + c_2 f_2(t)\} = c_1 \mathcal{L}\{f_1(t)\} + c_2 \mathcal{L}\{f_2(t)\}$$

The First Shift Theorem:

$$\mathcal{L}\left\{e^{-at} f(t)\right\} = F(s+a)$$

Derivative:

$$\mathcal{L}\left\{\frac{df}{dt}\right\} = \mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\left\{\frac{d^2 f}{dt^2}\right\} = \mathcal{L}\{f''(t)\} = s^2 F(s) - sf(0) - f'(0)$$

Short Table of Laplace Transforms

$f(t)$	$F(s) = \int_0^\infty e^{-st} f(t)dt$
k	$\frac{k}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$\sin bt$	$\frac{b}{s^2+b^2}$

Fourier Series Representation

If $f(t)$ is periodic with period T then

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2\pi nt}{T}\,dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2\pi nt}{T}\,dt$$

Fourier Series of the function f is then given by:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left\{a_n\cos\frac{2\pi nt}{T} + b_n\sin\frac{2\pi nt}{T}\right\}$$

Parseval's Theorem

If the function $f(t)$ is periodic with period T and has Fourier coefficients a_n and b_n then Parseval's Theorem states:

$$P_{av} = \frac{2}{T}\int_0^T [f(t)]^2\,dt = \frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

Amplitude Phase Representation

The amplitude-phase form of the Fourier representation of $f(t)$ is given by

$$f(t) = A_0 + \sum_{n=1}^{\infty}\Big[A_n\cos(nt - \phi_n)\Big]$$

where $A_0 = \frac{a_0}{2}$, $A_n = \sqrt{a_n^2 + b_n^2}$ and $\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

Discrete Frequency Spectra

A plot of

$$A_n \text{ versus } n$$

for each $n \in N$ is an **amplitude spectrum** of $f(t)$.

A plot of

$$\phi_n \text{ versus } n$$

for each $n \in N$ is a **phase spectrum** of $f(t)$.

Between them, these spectra constitute the **discrete frequency spectra.**

Integration by parts:

$$\int t \sin at \, dt = -\frac{t}{a} \cos at + \frac{1}{a^2} \sin at$$

$$\int t \cos at \, dt = \frac{t}{a} \sin at + \frac{1}{a^2} \cos at$$

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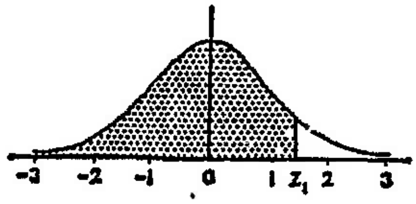
Area under the Normal Curve

$$P(z \leqslant z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2}\, dz .$$



z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0·1	0·5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
0·2	0·5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
0·3	0·6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
0·4	0·6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0·5	0·6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0·6	0·7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0·7	0·7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
0·8	0·7881	7910	7939	7967	7995	8023	8051	8078	8106	8133
0·9	0·8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1·0	0·8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1·1	0·8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1·2	0·8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1·3	0·9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1·4	0·9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1·5	0·9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1·6	0·9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1·7	0·9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1·8	0·9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1·9	0·9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2·0	0·9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2·1	0·9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2·2	0·9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2·3	0·9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2·4	0·9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2·5	0·99379	99396	99413	99430	99446	99461	99477	99492	99506	99520
2·6	0·99534	99547	99560	99573	99585	99598	99609	99621	99632	99643
2·7	0·99653	99664	99674	99683	99693	99702	99711	99720	99728	99736
2·8	0·99744	99752	99760	99767	99774	99781	99788	99795	99801	99807
2·9	0·99813	99819	99825	99831	99836	99841	99846	99851	99856	99861
3·0	0·99865	99869	99874	99878	99882	99886	99889	99893	99897	99900
3·1	0·99903	99906	99910	99913	99916	99918	99921	99924	99926	99929
3·2	0·99931	99934	99936	99938	99940	99942	99944	99946	99948	99950
3·3	0·99952	99953	99955	99957	99958	99960	99961	99962	99964	99965
3·4	0·99966	99968	99969	99970	99971	99972	99973	99974	99975	99976
3·5	0·99977	99978	99978	99979	99980	99981	99981	99982	99983	99983
3·6	0·99984	99985	99985	99986	99986	99987	99987	99988	99988	99989
3·7	0·99989	99990	99990	99990	99991	99991	99992	99992	99992	99992
3·8	0·99993	99993	99993	99994	99994	99994	99994	99995	99995	99995
3·9	0·99995	99995	99996	99996	99996	99996	99996	99996	99997	99997

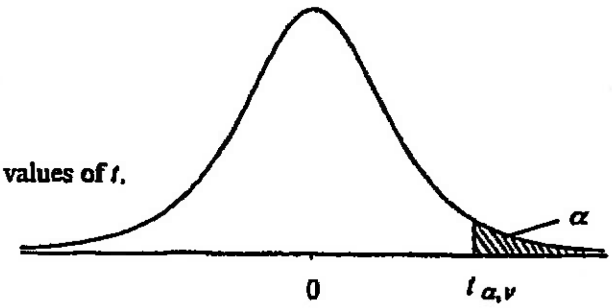
Table 7 Percentage Points of the t Distribution

The table gives the value of $t_{\alpha,\nu}$ – the 100α percentage point of the t distribution for ν degrees of freedom.

The values of t are obtained by solution of the equation:

$$\alpha = \Gamma[\tfrac{1}{2}(\nu+1)][\Gamma(\tfrac{1}{2}\nu)]^{-1}(\nu\pi)^{-1/2}\int_t^{\infty}(1+x^2/\nu)^{-(\nu+1)/2}\,dx$$

Note: The tabulation is for one tail only, that is, for positive values of t.
For $|t|$ the column headings for α should be doubled.



$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$\nu =$ 1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

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