CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/18

Module Title: Calculus and Statistical Analysis

Module Code: MATH8001

School: Science

Programme Titles: BSc (Hons) in Analytical Chemistry and Quality Assurance
BSc (Hons) in Environmental Science and Sustainable Technology

Programmes Codes: CR_SCHQA_8_Y2
CR_SESST_8_Y2

External Examiner: Dr. James Cruickshank

Internal Examiners: Ms. Patricia Cogan

Instructions: Answer ALL questions (each question carries equal marks)
Do not write, draw or underline in RED.
Show all calculations and workings in full.

Duration: 2 hours

Sitting: Autumn 2018

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination. If in doubt please contact an Invigilator.

Question 1

(a) Calculate the equation of the tangent line to the following curve at the point $(2,1)$.

$$x^2y^2 - 5y^3 = 2x - 5y$$

[12 marks]

(b) Calculate the partial derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ for the following:

$$Z = \frac{1}{x^2} - 6x^2y^4 + \frac{2x^3}{y^5}$$

[8 marks]

Question 2

(a) Calculate the following integrals:

(i) $\int_1^2 2x\, e^{3x-5}\, dx$

(ii) $\int \sin^3 x\, .\cos x\, dx$

[13 marks]

(b) A body moves from rest under the action of a direct force given by $F = \frac{25}{1-3s}$ Newtons, where s is the distance in metres from the starting point. Find the total work done in moving a distance of 0.25m. (i.e. from $s = 0$ to $s = 0.25$) if $W = \int F\, ds$.

[7 marks]

Question 3

(a) Two cards are selected from a standard deck without replacement. Calculate the probability of selecting:

(i) Two kings.

(ii) Exactly 1 king.

(iii) At least 1 king.

(iv) No kings

[10 marks]

(b) The protein content of a variety of parmesan cheeses is found to be normally distributed about a mean of 44 g per hundred grams and a standard deviation of 1.1 g. Determine the probability that for a cheese chosen at random, the protein content is

(i) Greater than 46 g.

(ii) Between 42.5 and 45 g.

[10 marks]

Question 4

An investigation was carried out to examine the linear relationship between the age of a driver and the distance the driver can see. The data collected is tabulated as follows:

Age of Driver (years) (x)	20	28	35	40	58	74
Distance (x100m) (y)	5.7	5.8	4.8	4.3	4.0	3.4

$$\sum x = 255\ , \sum y = \ 28\ , \sum xy = \ 1100\ , \sum x^2 = \ 12849\ , \sum y^2 = \ 135.22$$

(a) Plot the data on a scatter diagram using the graph paper in the centre of the exam booklet.

[4 marks]

(b) Calculate the least squares line of regression, rounding the values of a and b to two decimals. Graph this line on the scatter diagram from part (a).

[12 marks]

(c) Calculate the coefficient of correlation correct to two decimals. Interpret your results.

[4 marks]

Question 5

(a) A random sample of 120 first year students surveyed in a third level college, showed that 78 of them passed their examinations on the first attempt. Find a 99% confidence interval for the proportion of all first year students who pass their examinations on the first attempt. Interpret your result.

[10 marks]

(b) A company manufactures a cleaning reagent which is stated to contain 9.5% HCl. A sample of 50 analyses has a mean content of 8.9% with a standard deviation of 1.5%. Is the manufacturer's claim justified at the 1% significance level? Interpret your results.

[10 marks]

Statistical Formulae

Descriptive Statistics

Mean: $\bar{x} = \frac{\sum x}{n}$ Standard Deviation: $s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$

Regression and Correlation

$$y = a + bx$$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \qquad a = \frac{\sum y - b\sum x}{n}$$

$$\text{Coefficient of Correlation} = r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2} \cdot \sqrt{n\sum y^2 - (\sum y)^2}}$$

Probability Distributions

Binomial Distribution:

$$P(r,n) = {}^{n}C_r \, p^r \, q^{n-r}$$

Poisson Distribution:

$$P(r) = \frac{\lambda^r \, . e^{-\lambda}}{r!}$$ Note: $e = 2.718$ approximately

Normal Distribution (Standard Units):

$$z = \frac{x - \mu}{\sigma}$$

Sampling

Standard Error of the Mean $= \frac{s}{\sqrt{n}}$

Standard Error of the Proportion $= \sqrt{\frac{pq}{n}}$

$Z -$ score $= \frac{\bar{x} - \mu}{s/\sqrt{n}}$

Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$

Product Rule

$$y = uv$$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Quotient Rule

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chain Rule

$$y = u\big(v(x)\big)$$

$$\frac{dy}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$

Integration

$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1} \quad n \neq -1$
$\frac{1}{x}$	$\ln x$
e^x	e^x
e^{ax}	$\frac{1}{a}\, e^{ax}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$

Integration by Parts Formula:

$$\int u.dv = u.v - \int v.du$$

Other Relevant Formulae:

Linear Equation

$$y - y_1 = m(x - x_1)$$

Newton-Raphson Formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Simpson's Rule

$$A = \frac{s}{3}\,[\,(F + L) + 4E + 2R\,]$$

Standard Normal Probabilities

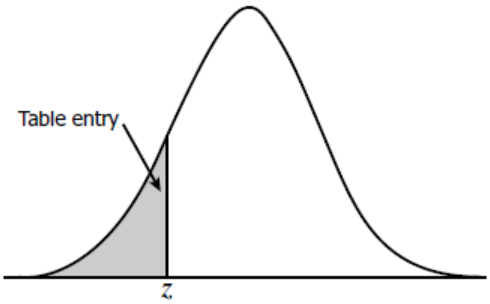

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

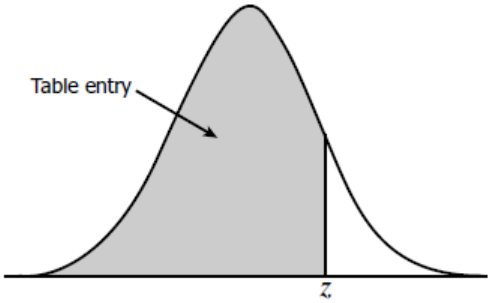

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998