CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/2018

Module Title: Engineering Mathematics 311

Module Code: MATH8003

School: School of Civil, Structural and Environmental Engineering

Programme Title: Bachelor of Engineering (Honours) in Structural Engineering

Programme Code: CSTRU_8_Y3

External Examiner(s): Dr Ann O'Shea

Internal Examiner(s): Dr Maryna Lishchynska

Instructions: Answer ALL questions.
Show all calculations in full.

Duration: 2 hours

Sitting: Autumn 2018

Requirements for this examination: Tables of Laplace Transforms, standard integrals, derivatives and Fourier formulae attached

Note to Candidates: Please check the **Programme Title** and the **Module Title** to ensure that you have received the correct examination.

If in doubt please contact an Invigilator.

Q1.

a) Find the inverse Laplace transform:

$$L^{-1}\left\{\left[\frac{32}{s(s^2+16)}\right]e^{-3s}\right\}$$

[5 marks]

b) Use Laplace transforms to solve the following differential equations

(i) $$\frac{d^2x}{dt^2}+4x=12\sin 2t \qquad x(0)=x'(0)=0$$

[9 marks]

(ii) $$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+3y=24\delta(t-4) \qquad y(0)=y'(0)=0$$

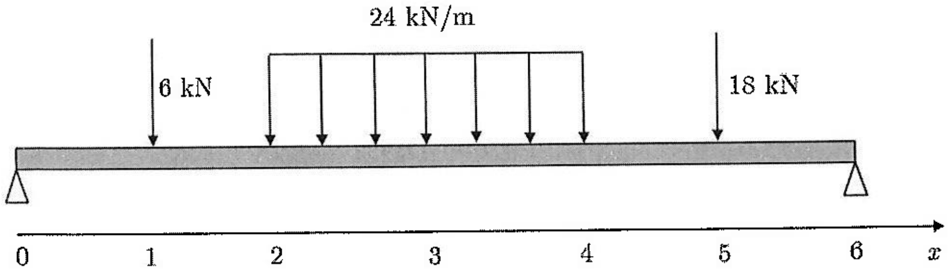
[6 marks]

c) Consider a 6 m long beam under the load shown in the figure below.

(i) Plot the load function $w(x)$ as a function of x.

(ii) Express the load function $w(x)$ in terms of unit step and unit impulse functions.

(iii) Find Laplace transform of the result from part (ii).



[4 marks]

P.T.O.

d) Consider a beam of span of 4 m fixed at both ends and x-axis aligned along the length of the beam. At $x = 2$ there is a concentrated load of 8 kN. The deflection of the beam at any point x along the beam length $y(x)$ is described by the differential equation:

$$EI \frac{d^4 y}{dx^4} = w(x).$$

(i) Express the beam load function $w(x)$ mathematically.

(ii) Express the boundary conditions mathematically.

(iii) Use Laplace Transforms to solve the equation for the beam deflection $y(x)$.

Note: $L\{f^{IV}(t)\} = s^4 F(s) - s^3 f(0) - s^2 f'(0) - sf''(0) - f'''(0)$

[11 marks]

P.T.O.

Q2.

a) By assuming an exponential solution find the general solution of the system of differential equations:

$$\frac{dx}{dt} = 7x - 2y + z$$
$$\frac{dy}{dt} = -2x + 10y - 2z$$
$$\frac{dz}{dt} = x - 2y + 7z$$

[15 marks]

b) In the theory of mechanical vibrations the system of differential equations below arises, where $x_1(t)$ and $x_2(t)$ are the displacements of two masses attached by two springs. By assuming periodic solutions of the form $R\cos(\omega t - \alpha)$ find a general solution of the system:

$$x_1'' = -13x_1 - 3x_2$$
$$x_2'' = -4x_1 - 12x_2$$

[10 marks]

P.T.O.

Q3.

a) Given

$$f(t) = \begin{cases} 2t & \text{if } 0 \le t < 1 \\ 4 - 2t & \text{if } 1 \le t \le 2 \end{cases}$$

Sketch this function and its even extension.

[3 marks]

b) A function $f(t)$ is defined as follows:

$$f(t) = \begin{cases} 4 & \text{if } 0 \le t < 4 \\ 0 & \text{if } 4 \le t \le 8 \end{cases} \qquad f(t+8) = f(t)$$

(i) Sketch two cycles of this function.

(ii) State whether this function is even, odd or neither.

(iii) Determine Fourier series of this function. Show terms as far as the 5^{th} harmonic inclusive.

[12 marks]

c) Suppose the heat is lost from the surface of a thin metal rod of length L into a surrounding medium at temperature zero. Then the heat equation takes on the form

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - hu$$

where h is a constant. Both ends of the rod are insulated i.e. $\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0$ and $\left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$ and the initial temperature distribution in the rod is $u(x,0)=f(x)$.

(i) Use substitution $u(x,t) = e^{-ht}\,v(x,t)$ to reduce the above problem to a homogeneous boundary value problem (BVP).

[6 marks]

P.T.O.

(ii) Use the method of separating the variables to solve the homogeneous BVP obtained in part (i).

[14 marks]

(iii) Show that the particular solution of the original non-homogeneous BVP when $f(x) = u_0$ is

$$u(x,t) = u_0 e^{-ht}$$

[5 marks]

Table of Laplace Transforms

$f(t)$	$F(s)$
a	$\frac{a}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin wt$	$\frac{w}{s^2+w^2}$
$\cos wt$	$\frac{s}{s^2+w^2}$
$\sinh kt$	$\frac{k}{s^2-k^2}$
$\cosh kt$	$\frac{s}{s^2-k^2}$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$e^{at}f(t)$	$F(s-a)$
$\int_0^t f(u)g(t-u)du$	$F(s)G(s)$
$u(t)$	$\frac{1}{s}$
$u(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$\delta(t)$	1
$\delta(t-a)$	e^{-as}

Trigonometric Identities

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$\sin^2 A = \frac{1}{2}(1-\cos 2A)$

$\cos^2 A = \frac{1}{2}(1+\cos 2A)$

$\cos(-A) = \cos A;\quad \sin(-A) = -\sin A$

Fourier Formulae

A	0	π	2π
$\sin A$	0	0	0
$\cos A$	1	-1	1

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2n\pi}{T}t\right) + b_n \sin\left(\frac{2n\pi}{T}t\right)\right]$$

$$a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt \ ; \qquad a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos\left(\frac{2n\pi}{T}t\right)dt \ ; \qquad b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin\left(\frac{2n\pi}{T}t\right)dt$$

Standard Integrals

$y = f(x)$	$\int f(x)\,dx$
x^n $(n \neq -1)$	$\frac{x^{n+1}}{n+1}$
e^{ax}	$\frac{1}{a}e^{ax}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\cos(ax+b)$	$\frac{1}{a}\sin(ax+b)$
$\sin(ax+b)$	$-\frac{1}{a}\cos(ax+b)$

$$\int x\sin ax\,dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}; \qquad \int x\cos ax\,dx = \frac{x\sin ax}{a} + \frac{\cos ax}{a^2}$$