CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/18

Module Title: Introduction to Statistics & Probability

Module Code: **STAT6010**

School: Mechanical, Electrical and Process Engineering

Programme Title: Bachelor of Engineering (Honours) in Sustainable Energy Engineering
Higher Certificate in Science in Good Manufacturing Practice and Technology
Bachelor of Engineering in Mechanical Engineering
Bachelor of Engineering in Biomedical Engineering

Programme Code: **CR_ESENT_8**
CR_SGMPR_6
CR_EMECH_7
CR_EBIME_7

External Examiner(s): **Prof. Michael Wallace**

Internal Examiner(s): **Ms. Jackie English, Ms. Katie Bullen**

Instructions: **Answer ALL questions.**
Each question carries 25 marks.
Total marks available: 100 marks.

Duration: **2 hours**

Sitting: Autumn 2018

Requirements for this examination: Graph paper

Please note that formulae and tables are included at the back of the examination paper. Candidates may also request a copy of Murdoch & Barnes *Statistical Tables* and/or the State Examinations Commission's *formulae and tables* booklet.

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

Question 1.

A sample of 100 electrical components was tested by operating each component continuously until it failed. The time to the nearest hour at which each component failed was recorded. The following results were obtained:

Time (hours)	Number of components
0 to less than 500	15
500 to less than 600	28
600 to less than 700	36
700 to less than 800	13
800 to less than 1000	8

(a) Calculate the mean and standard deviation of the time to failure. What does the standard deviation tell us about the data set? **[10 marks]**

(b) Use an appropriate formula to estimate the value of the median. What does this value represent? **[5 marks]**

(c) Using the graph paper at the centre of your exam booklet, draw a histogram for this distribution. **[6 marks]**

(d) Use an appropriate formula to calculate the mode. **[4 marks]**

Question 2.

(a) A and B are two events for an experiment, if $P(A) = {}^{3}/_{5}$ and $P(B) = {}^{11}/_{20}$

(i) Are A and B mutually exclusive events? Explain your answer.

(ii) If $P(A \cup B) = 0.9$, determine if A and B are independent events. **[4 marks]**

(b) Three cards are selected at random from a standard pack of cards, without replacement. What is the probability that the cards contain:

(i) Exactly one Jack

(ii) At least one Jack **[5 marks]**

(c) Ten points lie on the circumference of a circle, how many possible triangles can be drawn having these points as the vertices? **[2 marks]**

(d) Records show that, on average 7 out of 10 people recover from a particular disease after receiving a certain treatment. If 12 people are treated for this particular disease, find the probability that:

(i) Exactly 8 people will recover.

(ii) Less than 10 people will recover. **[7 marks]**

(e) The number of cars that a mechanic can service per day follows a Poisson distribution. If the mechanic can service on average five cars per eight hour day, what is the probability that the mechanic will service:

(i) Exactly five cars during any given day?

(ii) At least two cars in four hours? **[7 marks]**

Question 3.

(a) With the introduction of private test centres the waiting time for driving tests has fallen. The waiting times are found to be approximately normally distributed with a mean waiting time of 12.5 weeks and a standard deviation of 2.8 weeks.

Find the proportion of applicants who have to wait

(i) more than 14 weeks

(ii) between 10 and 14 weeks for a driving test.

(iii) How long will an applicant be waiting if they are among the 10% with the longest wait? **[10 marks]**

(b) Transaction times at a post office have a negative exponential distribution with a mean transaction time of 4 minutes.

Find the percentage of transaction times between 2 and 6 minutes. **[5 marks]**

(c) Given the data in the table below, approximate f (1.83) using a 3rd degree Newton-Gregory interpolating polynomial.

x_i	1.0	3.0	5.0	7.0	9.0
f_i	0	1.0	1.61	1.95	2.20

[10 marks]

Question 4.

(a) 65% of houses in a residential area have an alarm. It is found that 30% of the owners of the alarmed houses have a dog and 42% of the owners of the non-alarmed houses have a dog

A house is selected at random. Calculate the probability that the house has a dog.

[5 marks]

(b) The data in the table below relates the weekly maintenance cost (€) to the age (in months) of seven machines of similar type in a manufacturing company.

Age	5	10	15	20	30	50
Cost	190	240	250	300	335	300

For this data:

$$\sum x = 155\,;\ \sum y = 1615\,;\ \sum xy = 45275\,;\ \sum x^2 = 5275\,;\ \sum y^2 = 448425$$

(i) Plot the data on a scatter diagram. **[6 marks]**

(ii) Use the least squares method to find the regression equation of Cost on Age and plot this line on the scatter diagram in part (i) of this question. [**8 marks]**

(iii) Calculate the coefficient of correlation and comment on your answer **[5 marks]**

(iv) Predict the cost of maintenance on a machine that is 25 months old. **[1 mark]**

Formulae

Descriptive Statistics

Mean: $\bar{x} = \frac{\sum x}{n}$ or $\bar{x} = \frac{\sum fx}{\sum f}$

Mode = $L + \left(\frac{D_1}{D_1+D_2}\right) C$ or Mode = $L_M + C_M \left(\frac{f_M - f_{M-1}}{2f_M - (f_{M-1}+f_{M+1})}\right)$

Median = $L_M + \left(\frac{\left(\frac{n}{2} - F_{M-1}\right)}{f_M}\right) C_M$

Population standard deviation: $\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$ or $\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$

Population standard deviation: $\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$ or $\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$

Sample standard deviation: $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$ or $s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$

Sample standard deviation: $s = \sqrt{\frac{\sum fx^2 - \frac{(\sum fx)^2}{\sum f}}{f-1}}$

Coefficient of Skewness = $\frac{Mean - Mode}{Standard\ Deviation}$

Coefficient of Variation = $\frac{\sigma}{\bar{x}}$

Probability

Addition Law: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Law: $P(A \cap B) = P(A)P(B|A)$

Bayes' Theorem: $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$

Discrete Random Variables

$$E(X) = \sum x\, P(X = x)$$

$$E(X^2) = \sum x^2\, P(X = x)$$

$$V(X) = E(X^2) - [E(X)]^2$$

Binomial Distribution: $P(X = r) = C(n, r)p^r(1 - p)^{n-r}$

Poisson Distribution: $P(X = r) = \frac{e^{-m}\, m^r}{r!}$

Hypergeometric Distribution: $P(X = r) = \frac{C(M,r)C(N-M,n-r)}{C(N,n)}$

Continuous Random Variables

$$E(X) = \int_{-\infty}^{\infty} xf(x)\, dx$$

$$E(X^2) = \int x^2 f(x) dx$$

$$V(X) = E(X^2) - [E(X)]^2$$

Negative Exponential Distribution

$$f(x) = ae^{-ax}, \quad F(x) = 1 - e^{-ax}, \; R(x) = e^{-ax}, E(X) = \frac{1}{a}$$

Normal Distribution

X is $N(\mu, \sigma) \Rightarrow Z = \frac{X-\mu}{\sigma}$ is $N(0,1)$

Regression and Correlation

$$y' = ax + b$$

$$a = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$b = \frac{\sum y - a\sum x}{n}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left(n\sum x^2 - (\sum x)^2\right).\left(n\sum y^2 - (\sum y)^2\right)}}$$

Interpolation

The Lagrange interpolating polynomial of degree n that fits the $n+1$ data points $(x_0, f_0), (x_1, f_1), \ldots, (x_n, f_n)$ is

$$P_n(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} f_0$$

$$+ \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} f_1$$

$$+ \cdots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})} f_n$$

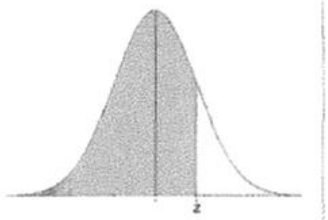
The Newton-Gregory interpolating polynomial of degree n evaluated at a specific x-value x_s can be written as

$$P_n(x_s) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_0 + \cdots$$

$$+ \frac{s(s-1)\ldots(s-n+1)}{n!}\Delta^n f_0$$

where $s = \frac{x_s - x_0}{h}$ and h is the uniform spacing between the x −values.

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998