CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2017/2018

Module Title: Technological Maths 302 (CA)

Module Code: STAT7003

School: School of Engineering

Programme Title: Mechanical Engineering – Year 3
Biomedical Engineering – Year 3

Programme Code: EMECH_7_Y3 EBIME_7_Y3

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External Examiner: Prof. Michael Wallace

Instructions:

- Answer all four questions in this booklet
- Question 1 to be answered in this booklet
- Do not write, draw or underline in RED.
- Show all calculations and workings in full.

Duration: 2 hours

Question1 **(30 Marks)**

Circle the correct answer in each of the following parts:

(i) Which of the following measures of central tendency is sensitive to extreme outliers?

(a) Mode
(b) Median,
(c) Mean,
(d) None of the above.

(3 Marks)

(ii) The data below show the time (in seconds) that a runner has beaten their personal best over their last 12 performances.

-6, -3, -3, 2, 1, 0, 3, 5, 12, 9, 12, 20

The median for the data is

(a) 0
(b) 4.3
(c) 2.5
(d) 3

(3 Marks)

(iii) An observation is considered an extreme outlier if it is:

(a) less than Q1−1.5 times the interquartile range
(b) less than Q3−1.5 times the interquartile range
(c) greater than Q1+1.5 times the interquartile range
(d) greater than Q3−1.5 times the interquartile range

(3 Marks)

(iv) Two dice are rolled and the sum is recorded. The probability of rolling a sum less than 5 is

(a) 0.167
(b) 0.278
(c) 0.833
(d) None of the above

(3 Marks)

(v) 65% of houses in a residential area have an alarm. It is found that 25% of the owners of the alarmed houses have a dog and 40% of the owners of the non-alarmed houses have a dog. A house is selected at random, the probability that the house does not have a dog is approximately:

(a) 0.065
(b) 0.303
(c) 0.698
(d) None of the above **(4 Marks)**

(vi) Twenty five samples of size 6 are taken from a stable process. The average mean of the sample means is 42.5, and the average range of the samples is 1.5.
What is the UCL and LCL for the $\bar{X}$-Chart?

(a) 44.0 and 41.0
(b) 43.37 and 41.63
(c) 43.22 and 41.78
(d) 47.5 and 37.5 **(4 Marks)**

(vii) The number of spelling mistakes in a report is denoted as the random variable X. The probability that there are at least 17 spelling mistakes in a report is given by

(a) $P(X < 17)$
(b) $P(X > 17)$
(c) $P(X \geq 17)$
(d) $P(X \leq 17)$ **(3 Marks)**

(viii) Assuming a normal distribution $P(-2 \leq z \leq 1.5)$ is equal to

(a) 0.9104
(b) 0.0896
(c) 0.044
(d) -0.9104

(3 Marks)

(ix) The college newsletter claims that the average CIT students play more sport than students from other colleges. The national college average is 6.2 hours per week, with a standard deviation of 1.2 hours. A sample of 40 college students play an average of 6.8 hours of sport per week. To carry out a hypothesis test on this claim we use a

(a) One-sided test and determine the z value
(b) One-sided test and determine the t-value
(c) Two-sided test and determine the z-value
(d) Two-sided test and determine the t-value.

(4 Marks)

Question 2 **(15 Marks)**

A survey asked students on average, how long they spend on social media sites. The results are as follows:

Number of Minutes	Frequency
0 but less than 15	14
15 but less than 30	36
30 but less than 45	54
45 but less than 60	71
60 but less than 75	85

(i) Construct a cumulative frequency table for the data

(ii) Represent this data on an Ogive, use graph paper

(iii)Estimate the Median from the Ogive.

Question 3 **(35 Marks)**

(a) A water analyst estimates that one in eight water samples tested per day have unacceptable levels of pollutant. What is the probability that among 15 samples analysed, there will be:
(i) exactly three samples with unacceptable levels of pollutant?
(ii) less than three samples with unacceptable levels of pollutant?

(b) The number of phone calls arriving at a helpdesk is modelled as a Poisson random variable. On average there are 10 calls per hour.
What is the probability that there are:
(i) exactly 5 calls in 1 hour?
(ii) 3 calls or less in 1 hour?

(c) The weight of food packed in containers has a Normal distribution with a mean weight of 300 grams and a standard deviation of 6 grams. Production requires that containers weigh 302 ± 10gr.
(i) What percentage of containers has weight below 294 grams?
(ii) Calculate the percentage of containers that is not within acceptable weight limits.
(iii)What weight can be set such that 95% of container weights are at least this value.

Question 4 **(20 Marks)**

It is suspected that there is some relationship between the relative humidity and the tensile strength of a certain material. The following measurements were obtained:

Relative Humidity, x (%)	Tensile Strength, y (N/m^2)
45	80
55	67
65	58
80	55
95	30

And $\sum \boldsymbol{x} = \mathbf{340}$, $\sum \boldsymbol{y} = \mathbf{290}$, $\sum \boldsymbol{xy} = \mathbf{18305}$, $\sum \boldsymbol{x^2} = \mathbf{24700}$, $\sum \boldsymbol{y^2} = \mathbf{18178}$

(a) Draw a scatter plot of the data (use graph paper)

(b) Calculate the correlation coefficient r, showing your workings, and interpret its value.

(c) Find the coefficient of determination, and comment on this result.

(d) Calculate the least squares regression line of y on .

(e) Hence estimate the tensile strength if the relative humidity is 60%.

Useful Formulae

Binomial distribution:

$$P(r,n) = {}^{n}C_r p^r q^{n-r} = \binom{n}{r} p^r q^{n-r}$$

Poisson distribution:

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!} = e^{-\lambda}\left(\frac{\lambda^r}{r!}\right)$$ Note: $e = 2.718$ approximately

Standard Normal Units:

$$Z = \frac{x - \mu}{\sigma}$$

Sampling standardising formulae:

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} \text{ or } z = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \text{ or } t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \text{ and } df = n-1$$

Correlation Coefficent: $r = \dfrac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$

Regression Line: $y = a + bx$, $b = \dfrac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$, and $a = \dfrac{\Sigma y}{n} - b\dfrac{\Sigma x}{n}$

Variable Control Charts: $UCL = \bar{\bar{x}} + A_2\bar{R}$ and $LCL = \bar{\bar{x}} - A_2\bar{R}$ or
$UCL = D_4\bar{R}$ and $LCL = D_3\bar{R}$

n	A_2	D_3	D_4	d_2
2	1.88	0	3.267	1.128
3	1.023	0	2.574	1.693
4	0.729	0	2.282	2.059
5	0.577	0	2.114	2.326
6	0.483	0	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.97
10	0.308	0.223	1.777	3.078
11	0.285	0.256	1.744	3.173
12	0.266	0.283	1.717	3.258
13	0.249	0.307	1.693	3.336
14	0.235	0.328	1.672	3.407
15	0.223	0.347	1.653	3.472

Normal Tables

Table A.3 Areas under the Normal Curve

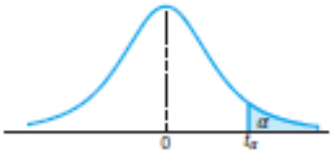
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Normal Tables continued

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table A.4 Critical Values of the t-Distribution



	α						
v	**0.40**	**0.30**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960